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Sophie Germain

Revolutionary Mathematician

Second Edition

 Springer

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Cover illustration: Sketch according to the bust of Sophie Germain by sculptor Zacharie Astruc (1835–1907). *Source* Stupuy (1896)

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*To my beloved mathematician,
for a lifetime of intellectual pleasure.*

Preface to the Second Edition

This new edition gives me the opportunity to expand on the mathematical work that Sophie Germain carried out, adding more details extracted from correspondence that had not previously been explained.

This edition will be distinguished from the previous one by the new material added and by a reorganization of the subjects. Chapter 3 is new. It focuses exclusively on the research on number theory that Sophie Germain carried out while communicating with Gauss in the years between 1804 and 1809. I added this chapter to highlight Germain's work on quadratic reciprocity before she matured her plan to prove Fermat's Last Theorem (FLT). Hence, the section in the original edition devoted to Germain's contribution to FLT has been redone almost entirely and now is addressed in Chap. 9.

Chapter 12 contains new details extracted from the letters written by Carl Bader to Gauss regarding Sophie Germain, and also from her last letter to Gauss in 1829, together with an explanation of Germain's last papers published in Crelle's Journal.

The remaining chapters have experienced enough changes to add clarity where needed. For example, I translated excerpts from correspondence and from published memoirs, historical pieces that are important to support the story.

In addition, this edition contains translations of nine of the 14 letters exchanged between Germain and Gauss. These epistles are supplemented by two other communications that help us shed light on the events that led to the revelation that the Monsieur Le Blanc corresponding with Gauss was actually Sophie Germain.

I also added a comprehensive list of References, including her own works and those of others, spanning from 1705 to the present.

The reader must be assured that I researched every detail about Sophie Germain's life and work. The citations in the footnotes are intended to validate every statement made throughout this book. I strived to avoid unsubstantiated claims or embellished stories, in order to portray a real mathematician with her

share of faults and intellectual triumphs. At the same time, it is my desire that this work will serve as inspiration for readers who may wish to continue exploring the historical archives to uncover more of Sophie Germain's endeavors.

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Preface to the First Edition

In the Age of Enlightenment, scholars were engaged in stimulating scientific work that led them to discover a myriad of novel mathematical methods. New branches of mathematics were born with those discoveries. The names of Euler, Bernoulli, Newton, Lagrange, Gauss, Legendre, Cauchy, and others are attached to these mathematical discoveries. Their efforts produced the powerful mathematical methods we apply today in all scientific disciplines so familiar to students of science and engineering.

Among those renown scholars we find the name of Sophie Germain, sometimes linked to Gauss, sometimes hyphenated to Lagrange's. As we learn more about her story, Sophie Germain emerges as an important figure that helped shape the foundation of two branches of mathematics: number theory and mathematical physics. A theorem on number theory related to the proof of Fermat's Theorem is named after Germain; today, we study her special proposition together with the results by Euler, Legendre, and other number theorists. Before Germain, no other woman had accomplished so much. Surprisingly, she never attended school! Despite her lack of scientific training and despite being excluded from the learned societies, Sophie Germain succeeded in developing her own theorems and in stirring new ideas in others.

Sophie Germain grew up in an era of mathematical and social revolution. Her childhood developed during the reign of the tragically infamous Louis XVI and Marie-Antoinette, and she came of age during the most tumultuous years of insurrection against the monarchy. Within months of her thirteenth birthday, the French Revolution exploded violently with the storming of the Bastille. Coincidentally, that same year Sophie awoke to the dawn of her own intellectual development, one that would convert her into an unusual and formidable mathematician. Alone, she undertook two of the most important problems of her time, a scientific feat that required incredible ingenuity in order to develop two diametrically different mathematical theories.

Sophie Germain was a mathematician, researcher, physicist, and philosopher. And thus, a book about her life must address diverse aspects of her work. I begin with her work in applied mathematics.

Sophie Germain was influential in the development of the theories of elasticity and vibrations. She derived the first equation that attempted to explain Chladni's vibrating plates. Overcoming prejudice and numerous obstacles to her scientific endeavors, in 1816 Sophie Germain became the first woman in the history of science to win the Prix de Mathématiques of the French Academy of Sciences. This prestigious contest was established at that time by the Premier Classe des Sciences Physiques et Mathématiques, a branch of the Institut de France, at the request of Napoléon.

The story of Sophie Germain's contribution to elasticity theory and the checkered history of her prize have already been told in Bucciarelli and Dworsky's fine book of 1980. In retelling it in chapter 6, I make use of my own historical research and also endeavor to give a different perspective on the mathematical ideas that sustained Sophie Germain to develop her own theory.

I continue to expound Germain's research through chapter 7, as I attempt to reconstruct the experiments with vibrating plates that she carried out after she won the prize. I will refer to her last publication to call attention to her experiments with vibrating plates. Seeing her engaged in such research activity suggests that Germain was not just a mathematician; she could very well be considered the first woman research engineer!

In number theory, Sophie Germain is the first and only woman to make a substantial contribution to the centuries-old proof of Fermat's Last Theorem. I will devote chapter 9 to her work in number theory, focusing especially on her theorem. For the most part, this chapter is based on the scholarly article of R. Laubenbacher and David Pengelley. I added additional historical and mathematical background to align their exposition to the context of this book.

Another aspect of Germain's work I review is her philosophy. This is important because I want the reader to hear Sophie's words, beautifully tinted by her deep love for mathematics and that reveal a bit more of her being. Finally, to better appreciate her background, I feel justified to include short biographical sketches of the mathematicians who helped shape her scientific legacy.

With many books and articles published about women mathematicians in history, biographical sketches of Sophie Germain abound. And because information about her childhood and educational beginning is obscure, some of those descriptions are often vague or inaccurate. Of course, Sophie Germain herself made it difficult for her biographers to ascertain the truth about the woman behind the fame. She left no autobiography or intimate letters that can shed light into her feelings about loved ones. Germain did not write about her childhood or her everyday life, focusing instead on describing her mathematical discoveries. Thus, a biographer has a daunting endeavor, trying to piece together what little historical evidence exists to ascertain Sophie Germain's private life with data about the development of science before and during her time.

I have a twofold aim in this book. I wish first to relate some historical developments in mathematical physics and in number theory, and in doing so, attempt to recreate the social and intellectual milieus in which Sophie Germain lived. Second, I hope to explain in easy terms the mathematical ideas and methods that Germain

pursued, and on which other contributions have been built. Most biographies of mathematicians are typically devoid of mathematics, focusing instead on details of their personal lives that make for a very attractive reading. I believe that there is room for a book like this, one that will go deeper into the history and the mathematical foundation of an intriguing woman who left us a beautiful scientific legacy.

Most of the citations in this book are taken from the originals; some I left in French to retain the subtlety or ambiguity of the expression, while other passages (unless otherwise noted) I translated myself.

My goal is to provide a perspective about the social and historical environment that surrounded Sophie Germain, while introducing the reader to some of her scientific endeavors. In many popular books, it is not uncommon to read embellished accounts of the lives of mathematicians. Disregarding the truth, many authors use apocryphal stories as fact, discrediting the biographical information they attempt to present. That leads to conveying to the reader many falsehoods that are repeated in other biographies and disseminated through the Internet.

I try to avoid unfounded speculation and only include in this book extracts from the life story of Sophie Germain that are fully documented. I verified historical facts, read her articles, and consulted many historical and scholarly publications. In addition to reviewing Germain's work, I also attempt to present a hint of the woman through an account of how life evolved in Paris, from her childhood, when France was besieged by its violent revolution, to the last years of her life. I hope that by including snippets of French history will increase our understanding of how Germain's life unfolded, and help us place in historical context her scientific isolation. I considered not only how Germain's work relates to the history of mathematics, but also as a means to reveal a glimpse of her temperament.

I did not confine this account to chronological order. Rather, I arranged it under different topics, a method which may result in some repetition or may lead me to present an event tinted by the clear eye of retrospect. However, the timeline at the end of the book presents a chronological sequence that may serve to guide us through the major events that shaped Sophie Germain's fifty-five-year-life. Chapter 11 interrupts the biographical narrative to provide short life-stories of the mentors, friends, and rivals, the erudite men who formed part of Germain's scientific story. Her own life story is remarkable and worth recounting, at least in part.

Of course, there are many questions about her that remain unanswered. As a young woman, Sophie Germain dazzled the Parisian intellectuals when they discovered that she carried out mathematical work in response to lectures by Lagrange at the *École Polytechnique*. How did Germain learn mathematics on her own before sending her analysis to Lagrange? Women were not admitted to the *École*. What drove Germain to circumvent the rules? She signed her work with the pseudonym M. Le Blanc; was this assumed name borrowed from a male student she knew? Who was Monsieur Le Blanc? Many have suggested that Lagrange became her teacher of mathematics. Then why did Lagrange not write about Germain to say how he discovered her? Lagrange did not talk about her talents nor did he describe her as a pupil. These and many other questions I raise in chapter 13, inquiries that may serve as the basis for further study.

The story of Sophie Germain is particularly important and inspiring because it reminds us of a time when higher education was not a right of women. Learning about her struggles to acquire an education, and to be part of the search for a better understanding of the world, may help us to appreciate the freedoms and opportunities we women enjoy now.

I believe this effort is important because it puts in perspective how a person pursues a scientific activity, including the learning that is required, and the tenacity and motivation scientists must have to pursue their work in spite of many obstacles and challenges.

This book is intended for anyone who wishes to learn more about Sophie Germain's work and the ideas she championed. Hers is a truly fascinating mathematical story. I hope that a wide range of readers will find it as interesting and exciting as I do, and this will encourage them to explore the beauty of mathematics in more detail and lead them to discover its many gems. Consequently, I have written neither a conventional biography nor a mathematical study meant solely for the benefit of specialists. In fact, my approach may be controversial to some.

At this point I want you to turn the page and begin to read Sophie Germain's life story. However, I must first acknowledge those who supported this endeavor, the two people who were aware of the first steps I took, and saw me stumbling in my effort to write a book that was accurate in its mathematics and also fair in its historical context.

First, I am sincerely indebted to Professor David Pengelley for reading the first drafts of chapter 9, and for his invaluable instruction regarding Germain's work on number theory. I also thank Professor Dominic Klyve wholeheartedly for the insightful comments he made on several aspects of the book. I ask them both for forgiveness if I didn't take their advice entirely. However, I will be forever grateful to David and Dominic for their encouragement and words of wisdom during the darkest hours when the feedback from others proved so disheartening.

I gratefully acknowledge the Bibliothèque Nationale de France for providing copies of many original materials, and especially for granting me permission to reproduce the sketches drawn by Sophie Germain and archived among her manuscripts.

Likewise, I acknowledge The Euler Archive where I found every one of Euler's publications and guided me through the historical referencing of the works in elasticity theory, sound and vibrating bodies, and a testament of Euler's contribution in number theory.

Finally, I thank my daughters Lauren and Dasi for their smart and thoughtful comments.

I hope you will find in this account a source of inspiration. It was an indescribable pleasure for me to write about Sophie Germain and her work, and it is my sincere desire that you will enjoy reading it, too.

Dora Musielak

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I owe an immense debt of gratitude to David Pengelley, Emeritus Professor of Mathematics at New Mexico State University. He read critically Chaps. 3 and 9 and provided invaluable comments, helping me to clarify the exposition of the topics in those chapters, especially the details of Sophie Germain's Theorem. His critique ensured that my retelling of what Germain did to advance Fermat's theorem is mathematically and historically correct and consistent with his scholarly findings.

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Finally, my eternal thanks to my daughters Dasein and Lauren, and to my husband Zdzislaw, for their loving support and knowing, *Omnes enim trahimur, et ducimur ad cognitionis et scientiae cupiditatem, in qua excellere pulchrum putamus.*

Dora Musielak

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