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Spectral Theory

Basic Concepts and Applications

Springer

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To Sarah

Preface

The plan for this book arose from the desire for an introductory text on spectral theory, which would not assume functional analysis as a prerequisite. I wanted this text to include applications involving the Laplacian operator, and yet be concise enough to be covered in a single semester. The inspiration comes in large part from requests for independent study projects from undergraduate or first-year graduate students. Many of these students have proposed topics in particular application areas of spectral theory, such as automorphic forms, differential geometry, or quantum mechanics. Although such applications are covered in sources such as Chavel [19] or Iwaniec [47], books at this level generally assume basic functional analysis and spectral theory as a prerequisite. Most of my independent study students have had some real and complex analysis, but not functional analysis. I wanted to have a text or short course that would bridge the gap between this background and more specialized topics.

Although there are many good introductory books on functional analysis, the shorter, one-semester texts generally do not include enough material on unbounded or differential operators for the applications I had in mind. Books that do cover this part of spectral theory comprehensively, such as the excellent series of Reed and Simon [69–72], are much longer and therefore less suitable for a quick introduction.

The present text thus represents my attempt to produce a short, accessible account of spectral theory that could serve as an introduction to a broad variety of application areas involving the Laplacian operator. It is primarily based on notes from a functional analysis course that I first gave about 15 years ago. To fit both the introductory theory and some interesting applications into one semester posed a significant challenge; it is already difficult to squeeze the essentials of functional analysis into a single term. My strategy was to focus on a relatively small list of applications (Weyl's law for bounded domains, the theory of Schrödinger operators with positive potentials, etc.). I built the first half of the course around these topics, limiting the coverage of functional analysis background to the material that was necessary for the chosen examples.

The outline for this book was developed by the same approach. The result is a treatment of functional analysis that differs from more traditional texts in two major ways. First, the focus is almost exclusively on separable Hilbert spaces, and much of the Banach space theory is omitted. Second, the theory of unbounded operators is developed from the beginning, rather than as an addendum to the bounded case. Applications to differential operators are introduced as early as possible, mainly in the examples.

After a brief historical introduction in Chapter 1, the main body of the text is roughly divided into two parts. Chapters 2 through 5 cover the theoretical background, from the theory of Hilbert spaces and unbounded operators to the proof of the spectral theorem. These chapters are sequential and strongly interdependent. The second part, consisting of Chapters 6 through 9, is devoted to more specific contexts such as the Dirichlet Laplacian or Schrödinger operators. These later chapters are essentially independent and could be read in any order. This structure provides the flexibility to support a one-semester functional course, a special topics course, or an independent study project in a particular application area.

The book is aimed at an advanced undergraduate or beginning graduate level. The reader is assumed to have background including real and complex analysis, measure theory, and linear algebra, but no previous knowledge of functional analysis. The necessary background material is sketched in the appendix, with references.

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