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Dražen Adamović · Paolo Papi  
Editors

# Affine, Vertex and W-algebras

 Springer

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# Preface

This volume is based on the INdAM Workshop “*Affine, Vertex and W-algebras*” held in Rome from 11 to 15 December 2017. This meeting was devoted to recent developments in the theory of vertex algebras, with particular emphasis on affine vertex algebras, affine  $W$ -algebras, and  $W$ -algebras appearing in physical theories such as logarithmic conformal field theory.

It is widely accepted in the mathematical community that the best way to study the representation theory of affine Kac–Moody algebras is by investigating the representation theory of the associated affine vertex and  $W$ -algebras. In this volume, this general idea can be seen at work from several points of view.

Most of the state-of-the-art topics dealt with at the INdAM Conference are represented in this volume, including

- The idea of *fusion*, which is developed through its connections with mathematical physics and with classical problems in vertex algebra theory (conformal embeddings);
- Relationships with finite-dimensional Lie theory (Kostant’s theory of Lie pairs, Vogan’s theory of the algebraic Dirac operator) and with Lie pseudoalgebras;
- Permutation orbifolds (in the case of fermionic vertex superalgebras);
- Higher Zhu algebras;
- Connections with combinatorics, which naturally arise in the construction of bases for standard representations of symplectic affine Lie algebras and for principal subspaces of generalized Verma modules;
- De Sole–Kac work on integrable Hamiltonian equations and Poisson vertex algebras, appearing through the notes of a postgraduate course held by De Sole.

Zagreb, Croatia  
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