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Krassimir T. Atanassov

Interval-Valued Intuitionistic Fuzzy Sets

 Springer

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*I dedicate this book to the 150th anniversary
of the establishment of the Bulgarian
Academy of Sciences—the oldest institution in
my home country.*

Preface

Intuitionistic Fuzzy Sets (IFSs) were introduced in 1983 as an extension of fuzzy sets of Lotfi Zadeh (1921–2017), and as soon as the first paper in which they were proposed, the justification of this extension was provided. Six years later, in 1989, Georgi Gargov (1947–1996) extended the IFSs to Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs). In the next years, the theory of ICIFS was enriched with new operations and operators, and covered in my 1999 monograph ‘Intuitionistic Fuzzy Sets: Theory and Applications’, published by Springer-Verlag.

I started working on the present book some 20 years ago, at the beginning of the century, but it was only the results obtained in the last 2 years that urged me to complete it. In this book, I present my main results and their applications which in my opinion are the ones most interesting. In the last decade, a number of predominantly applied studies on IVIFS have appeared. I would encourage their authors to follow my example and collect their results in books.

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I would like to express my deepest gratitude to my former Ph.D. students Velin Andonov and Peter Vassilev, and to my daughter Vassia Atanassova for their careful reading and correction of the manuscript.

Sofia, Bulgaria
June 2019

Krassimir T. Atanasov

Contents

1	On Brouwer’s Intuitionism and Intuitionistic Fuzziness	1
	References	7
2	On Interval Valued Intuitionistic Fuzzy Sets	9
2.1	Intuitionistic Fuzzy Sets and Interval Valued Fuzzy Sets	9
2.2	Definition of Interval Valued Intuitionistic Fuzzy Sets	10
2.3	On Representability of Interval Valued Intuitionistic Fuzzy Sets by Pairs of Intuitionistic Fuzzy Sets	14
2.4	Some Ways for Altering Experts’ Estimations	16
2.5	Norms and Metrics on IVIFS	22
	References	24
3	Relations and Operations over IVIFSs	27
3.1	Relations over IVIFSs	27
3.2	Operations over IVIFSs	28
3.3	An IVIFS Whose Universe is an IFS or an IVIFS with Respect to Another Universe	43
3.4	IVIFSs over Different Universes	46
3.5	The Extension Principle for the IVIFS-Case	47
	References	51
4	Operators over IVIFSs	53
4.1	Interval Valued Intuitionistic Fuzzy Modal Operators of First Type	53
4.2	Interval Valued Intuitionistic Fuzzy Modal Operators of Second Type	69
4.3	Theorem for Equivalence of the Two Most Extended Modal Operators	83
4.4	Topological Operators over IVIFSs	89
4.5	Level and Modal-Level Operators on IVIFSs	96
4.6	Level Operator That Decrease the Number of Elements of the IVIFSs	107

4.7	Two Other Types of Operators	110
4.8	Simplest and (α, β) -Shrinking Operators over IVIFSs	116
4.9	Weight-Center Operator	120
	References	122
5	Interval Valued Intuitionistic Fuzzy Pairs	123
5.1	Definition of an IVIFP	123
5.2	Relations over IVIFPs	123
5.3	Operations over IVIFPs	124
5.4	Operators over IVIFPs	125
5.5	On IVIF-Interpretation of Interval Data	128
	References	129
6	Applications of IVIFSs	131
6.1	Extended Interval Valued Intuitionistic Fuzzy Index Matrices	132
6.2	Interval Valued Intuitionistic Fuzzy Graphs	140
6.3	Intuitionistic Fuzzy Neural Networks with Interval Valued Intuitionistic Fuzzy Conditions	147
6.4	Interval Valued Intuitionistic Fuzzy Generalized Nets	149
6.5	Intercriteria Analysis with Interval-Valued Intuitionistic Fuzzy Evaluations	164
6.6	Interval Valued Intuitionistic Fuzzy Sets as Tools for Evaluation of Data Mining Processes—Possibilities for the Future	170
6.6.1	IVIF-Estimations in Expert Systems, Data Bases, Data Warehouses, Big Data, OLAP-Structures	171
6.6.2	IVIF-Estimations of a Procedure for Inductive Reasoning	172
6.6.3	IVIF-Estimations in Decision Making Procedures	174
6.6.4	IVIFS-Estimations in Pattern Recognition Procedures	174
6.6.5	IF-Estimations in Neural Networks and Evolutionary Algorithms	176
	References	178
7	Conclusion and Remarks on Future Research	195
	References	197
	Index	199

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He has been serving as E-i-C of different journals. He is the author of numerous monographs, including the book *Intuitionistic Fuzzy Logics* (Springer, 2017). He is a corresponding member of the Bulgarian Academy of Sciences.