Graduate Texts in Mathematics 149

Graduate Texts in Mathematics

Graduate Texts in Mathematics bridge the gap between passive study and creative understanding, offering graduate-level introductions to advanced topics in mathematics. The volumes are carefully written as teaching aids and highlight characteristic features of the theory. Although these books are frequently used as textbooks in graduate courses, they are also suitable for individual study.

More information about this series at http://www.springer.com/series/136

John G. Ratcliffe

Foundations of Hyperbolic Manifolds

Third Edition

Springer

John G. Ratcliffe
Department of Mathematics
Vanderbilt University
Nashville, TN, USA

ISSN 0072-5285 ISSN 2197-5612 (electronic)
Graduate Texts in Mathematics
ISBN 978-3-030-31596-2 ISBN 978-3-030-31597-9 (eBook)
https://doi.org/10.1007/978-3-030-31597-9

Mathematics Subject Classification (2010): 57M50, 20H10, 30F40

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To Susan, Kimberly, Thomas, and Jack

Preface

This book is an exposition of the theoretical foundations of hyperbolic manifolds. It is intended to be used both as a textbook and as a reference. Particular emphasis has been placed on readability and completeness of argument. The treatment of the material is for the most part elementary and self-contained. The reader is assumed to have a basic knowledge of algebra and topology at the first-year graduate level of an American university.

The book is divided into three parts. The first part, consisting of Chapters 1-7, is concerned with hyperbolic geometry and basic properties of discrete groups of isometries of hyperbolic space. The main results are the existence theorem for discrete reflection groups, the Bieberbach theorems, and Selberg's lemma. The second part, consisting of Chapters 8-12, is devoted to the theory of hyperbolic manifolds. The main results are Mostow's rigidity theorem and the determination of the structure of geometrically finite hyperbolic manifolds. The third part, consisting of Chapter 13, integrates the first two parts in a development of the theory of hyperbolic orbifolds. The main results are the construction of the universal orbifold covering space and Poincaré's fundamental polyhedron theorem.

This book was written as a textbook for a one-year course. Chapters 1-7 can be covered in one semester, and selected topics from Chapters 8-12 can be covered in the second semester. For a one-semester course on hyperbolic manifolds, the first two sections of Chapter 1 and selected topics from Chapters 8-12 are recommended. Since complete arguments are given in the text, the instructor should try to cover the material as quickly as possible by summarizing the basic ideas and drawing lots of pictures. If all the details are covered, there is probably enough material in this book for a two-year sequence of courses.

There are over 500 exercises in this book, which should be read as part of the text. These exercises range in difficulty from elementary to moderately difficult, with the more difficult ones occurring toward the end of each set of exercises. There is much to be gained by working on these exercises.

An honest effort has been made to give references to the original published sources of the material in this book. Most of these original papers are well worth reading. The references are collected at the end of each chapter in the section on historical notes.

This book is a complete revision of my lecture notes for a one-year course on hyperbolic manifolds that I gave at the University of Illinois during 1984.

I wish to express my gratitude to:

(1) James Cannon for allowing me to attend his course on Kleinian groups at the University of Wisconsin during the fall of 1980;

(2) William Thurston for allowing me to attend his course on hyperbolic 3-manifolds at Princeton University during the academic year 1981-1982 and for allowing me to include his unpublished material on hyperbolic Dehn surgery in Chapter 10;

(3) my colleagues at the University of Illinois who attended my course on hyperbolic manifolds, Kenneth Appel, Richard Bishop, Robert Craggs, George Francis, Mary-Elizabeth Hamstrom, and Joseph Rotman, for their many valuable comments and observations;

(4) my colleagues at Vanderbilt University who attended my ongoing seminar on hyperbolic geometry over the last seven years, Mark Baker, Bruce Hughes, Christine Kinsey, Michael Mihalik, Efstratios Prassidis, Barry Spieler, and Steven Tschantz, for their many valuable observations and suggestions;

(5) my colleagues and friends, William Abikoff, Colin Adams, Boris Apanasov, Richard Arenstorf, William Harvey, Linda Keen, Ruth Kellerhals, Victor Klee, Bernard Maskit, Hans Munkholm, Walter Neumann, Alan Reid, Robert Riley, Richard Skora, John Stillwell, Perry Susskind, and Jeffrey Weeks, for their helpful conversations and correspondence;

(6) the library staff at Vanderbilt University for helping me find the references for this book;

(7) Ruby Moore for typing up my manuscript;

(8) the editorial staff at Springer-Verlag New York for the careful editing of this book.

I especially wish to thank my colleague, Steven Tschantz, for helping me to prepare this book on my computer and for drawing most of the 3-dimensional figures on his computer.

Finally, I would like to encourage readers to send me their comments and corrections concerning the text, exercises, and historical notes.

Nashville, June, 1994 JOHN G. RATCLIFFE

Preface to the Third Edition

The third edition includes hundreds of changes and additions, including over 40 new lemmas, theorems, and corollaries. The following theorems are new in the third edition: 7.5.3, 7.5.4, 7.5.7, 7.6.8, 12.3.7, 12.4.8, 12.4.9, 12.4.10, 12.4.11, 12.6.6, 12.6.7, 12.8.1, 12.8.2, 12.8.3, 12.8.4, 12.8.5, 12.8.6, 12.8.7, 12.8.8, 12.8.9, 12.8.10, 12.8.11, 12.8.12, 13.2.5, 13.2.7, 13.2.8, 13.2.9, 13.2.10, and 13.2.11. Moreover, Theorems 6.8.7 and 8.1.5 have been enhanced, and Theorems 3.5.6, 4.4.3, 6.3.2, and 9.4.3 have new proofs.

The notation for the positive Lorentz group has changed from $\mathrm{PO}(n,1)$ to $\mathrm{O}^+(n,1)$ and $\mathrm{PO}(n,1)$ is now defined to be the projective Lorentz group $\mathrm{O}(n,1)/\{\pm I\}$. The new notation is more consistent with standard practice.

The following are the major changes in the third edition. Almost all of the figures have been rendered in color. The most important elements of a figure are usually colored red. An introduction to enhanced Coxeter graphs for hyperbolic Coxeter groups has been added to §7.1. A more thorough development of the theory of crystallographic groups is now given in §7.5. More finiteness properties of geometrically finite groups have been added to Sections §12.4 and §12.6. The most significant change in the third edition is the addition of §12.8 on arithmetic hyperbolic groups. This section was originally planned for the first edition. Finally, more theory on compact geometric orbifolds has been added to §13.2, including an introduction to 2-dimensional geometric orbifolds.

To make room for all the new material in the third edition, the historical notes have been condensed by removing the titles of books and papers. For the reader of an e-version of the third edition, this hardly matters, since the book is now hyperlinked. The references have been pruned and updated and now have links back to the historical sections in which they are cited.

There are over 70 new exercises. The exercise sets for the important Sections §3.5 and §4.7 have doubled in size. Solutions to all the exercises in the third edition will be made available in a solution manual.

Finally, I wish to express my thanks to my colleague Steven Tschantz for major help on technical aspects of the production of this book.

Nashville, August, 2019 John G. Ratcliffe

Contents