

# Analog/Digital Implementation of Fractional Order Chaotic Circuits and Applications

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 Springer

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# Preface

Nowadays, noninteger or fractional-order chaotic systems are very interesting topics to engineers, physicists, and mathematicians because most real physical systems are inherently nonlinear in nature. It is worth mentioning that several books have recently been published detailing implementations of integer-order chaotic and hyperchaotic circuits and systems using analog integrated circuits technology, discrete devices, field-programmable analog arrays (FPAAs), and digital hardware like micro-controllers, field-programmable gate arrays (FPGAs), and multi-core systems. However, this is not the case for fractional-order chaotic systems, which mathematical models consist of fractional derivatives in nonlinear equations that are difficult to be solved by analytical methods.

Recent works have demonstrated that fractional-order differential equations are, at least, as stable as their integer-order counterpart. So that one can study a fractional-order mathematical model of a chaotic system to evaluate their equilibrium points, stability, and perform numerical simulation applying appropriate methods in the frequency and time domains, such as Caputo's definition, Grünwald–Letnikov method, and Adams–Bashforth–Moulton method. In addition, and under certain conditions, the dynamics analyses required by fractional-order chaotic systems can be performed in the same way as those done for integer-order ones, e.g., evaluation of the Lyapunov exponents, entropy, and fractal dimension.

Integer-order chaotic systems are relatively simple to implement with analog and digital hardware. However, analog implementations suffer the sensitivity problem of the analog component values and digital implementations suffer the problem of degradation due to the reduced number of bits to perform computer arithmetic operations. These problems are more difficult to solve in implementing fractional-order chaotic systems, which require more complex analog circuitry to solve fractional derivatives, thus increasing variations in the circuit elements; and require more complex digital hardware to implement memory blocks and control blocks that are required by time simulation methods. In this manner, it can be appreciated that the main objective of recent and related scientific works are oriented to introduce alternatives for the analog/digital implementation of fractional-order chaotic oscillators, trying to provide details on the synthesis and physical realization

using either analog or digital electronics. That way, this book provides guidelines for analog and digital implementations of fractional-order chaotic systems, which are performed from applying numerical methods in the frequency and time domains, to approximate the solutions and to synthesize the mathematical descriptions using amplifiers, FPAA's and FPGAs. The electronic implementations are measured in laboratory conditions to observe experimental fractional-order chaotic attractors, which are used to implement random number generators and secure communication systems for image encryption.

The contents of this book are organized in seven chapters devoted to describe the basic theory and review of mathematical models of several fractional-order chaotic systems that are simulated with different numerical methods; a review on the implementation of integer-/fractional-order chaotic oscillators using analog/digital hardware; characterization of the Lyapunov exponents, Kaplan–Yorke dimension and entropy, and their optimization applying metaheuristics; guidelines to approximate fractional derivatives in the frequency domain and its implementation using amplifiers and FPAA's; guidelines to perform VHDL descriptions of different fractional-order chaotic oscillators, which can be implemented on FPGAs, and guidelines to reduce hardware resources; and details to implement random number generators and the synchronization of fractional-order chaotic oscillators applying different techniques to develop applications in chaos-based secure communications and image encryption. The book ends with a discussion of recent and future trends on this hot topic from fractional calculus and highlights alternatives on the electronic implementation of fractional-order chaotic circuits and systems to enhance modern engineering applications in the era of hardware security for Internet of Things.

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# Contents

<b>1</b>	<b>Integer and Fractional-Order Chaotic Circuits and Systems</b> .....	1
1.1	Chaotic Circuits and Systems .....	1
1.2	Equilibrium Points and Eigenvalues .....	3
1.3	Numerical Methods for Integer-Order Chaotic Oscillators .....	4
1.3.1	One-Step Methods: Forward Euler and Runge-Kutta .....	5
1.3.2	Multistep Methods: Adams-Bashforth and Adams-Moulton .....	6
1.3.3	Comparison of One-Step and Multistep Numerical Methods .....	7
1.4	Numerical Simulation of Integer-Order Chaotic Oscillators .....	10
1.5	Lyapunov Exponents, Kaplan-Yorke Dimension, and Entropy .....	18
1.6	Fractional-Order Chaotic Systems .....	22
1.7	Definitions of Fractional-Order Derivatives and Integrals .....	24
1.7.1	Grünwald-Letnikov Fractional Integrals and Derivatives .....	25
1.7.2	Riemann-Liouville Fractional Integrals and Derivatives .....	27
1.7.3	Caputo Fractional Derivatives .....	27
1.8	Numerical Methods for Fractional-Order Chaotic Oscillators .....	28
1.9	Simulation of the Fractional-Order Derivative ${}_0D_t^q y(t) = x(t)$ .....	32
1.9.1	Approximation Applying Grünwald-Letnikov Method .....	33
1.9.2	FDE12 Predictor-Corrector Method .....	34
1.9.3	Adams-Bashforth-Moulton Method .....	35
	References .....	37
<b>2</b>	<b>FPAA-Based Implementation and Behavioral Descriptions of Autonomous Chaotic Oscillators</b> .....	41
2.1	FPAA-Based Implementation of Autonomous Chaotic Oscillators .....	41
2.1.1	Implementation Using AN231E04 FPAA .....	44
2.2	Simulating Chaotic Oscillators in Anadigm Designer Tool .....	49
2.3	Implementing Chaotic Oscillator Using FPGAs .....	54
2.4	VHDL Descriptions for Implementing Chaotic Oscillators .....	59



2.5 Experimental Observation of Chaotic Attractors Truncating the Fixed-Point Representation ..... 69

References ..... 74

**3 Characterization and Optimization of Fractional-Order Chaotic Systems ..... 75**

3.1 Fractional-Order Chaotic Oscillators..... 75

3.2 Computing Lyapunov Exponents Applying Wolf’s Method and TISEAN..... 77

    3.2.1 Evaluating Lyapunov Exponents by Wolf’s Method..... 77

    3.2.2 The Lyapunov Spectrum Provided by TISEAN..... 80

3.3 Optimizing Fractional-Order Chaotic Oscillators Applying DE and PSO Algorithms..... 82

    3.3.1 Differential Evolution Algorithm ..... 84

    3.3.2 Particle Swarm Optimization Algorithm ..... 84

3.4 Optimizing the Positive Lyapunov Exponent and Kaplan–Yorke Dimension Applying Metaheuristics ..... 86

References ..... 90

**4 Analog Implementations of Fractional-Order Chaotic Systems ..... 93**

4.1 Implementation of Fractional-Order Chaotic Oscillators Using Amplifiers and Fractors ..... 93

4.2 Ladder Fractance to Implement the Fractional-Order Lorenz Chaotic Circuit ..... 98

4.3 Tree Fractance to Implement the Fractional-Order Lorenz Chaotic Circuit ..... 101

4.4 Implementation of Fractional-Order Chaotic Oscillators Using FPAA’s ..... 105

References ..... 113

**5 FPGA-Based Implementations of Fractional-Order Chaotic Systems ..... 115**

5.1 On Grünwald-Letnikov Method ..... 115

5.2 Effects of the Short Memory Principle..... 119

5.3 Hardware Descriptions in VHDL ..... 124

    5.3.1 Library Package..... 125

    5.3.2 Entity Declaration ..... 130

    5.3.3 Architecture ..... 132

5.4 Computer Arithmetic ..... 134

    5.4.1 Example: Fixed-Point Representation of  $-2.75$  ..... 135

5.5 VHDL Description of Some Most Used Digital Blocks ..... 136

    5.5.1 Adder and Subtractor ..... 136

    5.5.2 Multiplier..... 138

    5.5.3 Multiplexer ..... 139

    5.5.4 Counter ..... 141

    5.5.5 Flip-Flop ..... 142

    5.5.6 Cumulative Summation Block ..... 143

- 5.6 Arrays for Designing Memories ..... 145
  - 5.6.1 Shift Register ..... 145
  - 5.6.2 Random Access Memories ..... 147
  - 5.6.3 Read Only Memory ..... 148
  - 5.6.4 Lookup Table ..... 149
- 5.7 FPGA-Based Implementation of Grünwald-Letnikov Method ..... 150
  - 5.7.1 FPGA-Based Implementation of Lorenz Fractional-Order Chaotic Oscillator Using RAMs and Shift Registers ..... 151
  - 5.7.2 Experimental Observation of Fractional-Order Chaotic Attractors ..... 153
- 5.8 FPGA-Based Implementation of Adams-Bashforth-Moulton Method ..... 160
  - 5.8.1 Simulating Fractional-Order Chaotic Oscillators Applying Adams-Bashforth-Moulton Approximation ..... 162
  - 5.8.2 Effects of the Short Memory Principle in Adams-Bashforth-Moulton Method ..... 162
  - 5.8.3 On the FPGA-Based Implementation of Adams-Bashforth-Moulton Method ..... 165
- References ..... 172
- 6 Synchronization and Applications of Fractional-Order Chaotic Systems ..... 175**
  - 6.1 Synchronizing Chaotic Oscillators in a Master-Slave Topology ..... 175
    - 6.1.1 Synchronizing Chaotic Oscillators in a “Master-Slave” Topology with Hamiltonian Forms and Observer Approach ..... 176
    - 6.1.2 Synchronizing Chaotic Oscillators in a “Master-Slave” Topology with OPCL ..... 177
  - 6.2 Synchronizing Fractional-Order Chaotic Oscillators ..... 178
    - 6.2.1 Hamiltonian Forms: Synchronization of Two Fractional-Order Lorenz Systems ..... 178
    - 6.2.2 OPCL: Synchronization of Two Fractional-Order Lorenz Systems ..... 179
    - 6.2.3 Hamiltonian Forms: Synchronization of Two Fractional-Order Rössler Systems ..... 183
    - 6.2.4 OPCL: Synchronization of Two Fractional-Order Rössler Systems ..... 188
  - 6.3 Fractional-Order Chaotic Secure Communication System Applied to Image Encryption ..... 191
  - 6.4 FPGA-Based Implementation of a Secure Communication System Based on Fractional-Order Chaotic Oscillators ..... 194
  - 6.5 Fractional-Order Chaos-Based Random Number Generators ..... 197
  - References ..... 201

**7 Conclusions and Issues for Future Research**..... 203  
    References ..... 206

**Index**..... 207

# Acronyms

CAB	Configurable Analog Block
CAM	Configurable analog module
CLB	Configurable logic block
CLK	Clock
CMOS	Complementary metal-oxide-semiconductor
$D_{KY}$	Kaplan–Yorke dimension
DAC	Digital-to-analog converter
DE	Differential evolution
DSP	Digital signal processor
EDA	Electronic design automation
EP	Equilibrium point
FE	Forward Euler
FPAA	Field-programmable analog array
FPGA	Field-programmable gate array
FSM	Finite state machine
$h$	Step-size
IEEE	Institute of Electrical and Electronics Engineers
IoT	Internet of Things
IP	Intellectual property
LE+	Positive Lyapunov exponent
LUT	Lookup table
MatLab	Matrix Laboratory
ODE	Ordinary differential equation
OpAmp	Operational amplifier
OPCL	Open-Plus-Closed-Loop
OTA	Operational transconductance amplifier
PID	Proportional-integral-derivative
PSO	Particle swarm optimization
PVT	Process-Voltage-Temperature
PWL	Piecewise-linear
RAM	Random access memory

RK	Runge–Kutta
ROM	Read only memory
RST	Reset
SCM	Single constant multiplier
SNLF	Saturated nonlinear function
TISEAN	Time Series Analyzer
VHDL	Very-high-speed-integrated-circuit hardware description language