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Discrete-Time and Discrete-Space Dynamical Systems

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Preface

Discrete-time and discrete-space dynamical systems are widely used in various areas, e.g., decision-making or computation. For instance, Boolean control networks were initially proposed to model genetic regulatory networks; finite automata and labeled Petri nets as models of discrete-event systems have been applied to many engineering fields such as manufacturing processes, production scheduling; nondeterministic finite-transition systems have been applied to model checking and automated synthesis of cyber-physical systems; cellular automata have been used in quantum physics, biological dynamics as well as computational mathematics, just to name a few.

Let us introduce what discrete-time and discrete-space dynamical systems mean in this book (transitioned from the well-known dynamical systems over Euclidean spaces). It seems widely accepted nowadays that the world consists of time elapsing "continuously" and space arranged "seamlessly". Evolution of a process within this setting can be described by nonlinear differential equations in a (locally) Euclidean space. Such description is intuitive but explicit solutions are not easy to obtain, which makes it difficult to analyze their long-term behavior. To bypass the obstacle of finding an explicit solution, one can discretize time to generate sequences of points (called *trajectories*) by iterations of maps. Another difficulty of analyzing long-term behavior of a dynamical system over a continuous state space lies in its continuity, since it is almost impossible to separate the crucial locations from the redundant ones. To overcome this dilemma, space is also discretized to guarantee *every sequence has a convergent subsequence*. For example, this property holds in any finite metric space; in addition, every sequence has an increasing subsequence (and thus convergent (possibly to infinity)) in the countable metric space $\mathbb{N}^n$ (as a subspace of $\mathbb{R}^n$); this property even holds in some uncountable spaces, like the Cantor space, where every point can be regarded as a mapping from $\mathbb{Z}^n$ to a common alphabet. It is worth mentioning that none of the three kinds of spaces above is locally Euclidean; indeed, they have topological dimension 0 (in the sense of Čech–Lebesgue covering dimension) while Euclidean spaces or general manifolds that are not the singleton have positive topological dimensions. For dynamical systems over these zero-dimensional spaces, explicit solutions can be found without any restriction on the system maps and therefore the focus is on the spaces. In this book, such dynamical systems are called discrete-time and discrete-space dynamical systems. What makes it challenging to study these systems mainly lies in the unrestricted system maps.

Among diverse fundamental properties (controllability, observability, detectability, and stabilizability) in control theory, observability and detectability stand out: both of them deduce internal information out of external information. Compared to controllability and stabilizability which focus directly on trajectories, observability and detectability are somewhat *indirect*; nevertheless, the latter provide basis for quantitative analysis of long-term behavior as well as prerequisites for (automated) controller synthesis.

Exploring long-term behavior is a long-lasting topic in dynamical systems. In this book, we will study decidability and complexity of observability and detectability as well as their variants for different kinds of discrete-time and discrete-space dynamical systems, such as the aforementioned Boolean control networks, finite automata, and labeled Petri nets. Various methods will be introduced to study different kinds of systems. Fundamental characterization for observability and detectability will (1) provoke new related studies such as state estimation and automated synthesis; (2) help reveal relations and essential differences among different types of systems. For instance, for finite automata, strong detectability is verifiable in polynomial time while weak detectability is PSPACE-complete; in contrast for labeled Petri nets, strong detectability is EXPSPACE-hard while weak detectability is even undecidable.

We will investigate another "indirect" property that is called "invertibility" or "reversibility". It means for control systems that an output sequence allows uniquely determining the corresponding input sequence, but it means for dynamical systems without control that every trajectory has a unique backward-in-time extension. For Boolean control networks, a tool (equivalent to cellular automata) beyond them is demanding to characterize invertibility, which in turn reveals the importance of invertibility in unveiling some link between Boolean control networks and cellular automata.

Finally, let us point out several potential applications of fundamental properties of discrete-time and discrete-space dynamical systems (particularly of such systems with finitely many states) in formal verification and synthesis of hybrid (control) systems. Since the verification problem for basic properties such as controllability and observability of (infinite-state) hybrid systems is mostly formidable and it is likely that many properties are undecidable. By constructing a finite-state system as an approximation which (bi)simulates a given hybrid system while preserving some useful property, one can benefit in verifying this property over the finite approximating system instead. Automated synthesis can be dealt with in an analogous way. From this perspective, results presented in this book can also be related to the active field of formal verification and synthesis emerging in the last two decades.

This book will first introduce (in Chap. 1) basic mathematical preliminaries such as graph theory, the semitensor product of matrices, finite automata, and topology to support the study throughout the book, and also the differences between different types of zero-dimensional spaces and Euclidean spaces based on these preliminaries. Second, it will discuss (in Chap. 2) different types of discrete-time and discrete-space dynamical systems (Boolean control networks, finite automata, nondeterministic finite-transition systems, Petri nets, and cellular automata), highlighting their similarities and differences. These essential differences show that there exists no unified method available to deal with these types of dynamical systems. Then in the main parts (the remaining chapters), it will collect a series of recent fundamental results in control-theoretic and topological dynamical problems of discrete-time and discrete-space dynamical systems, e.g., invertibility, observability, detectability, reversibility, etc., by developing new techniques. In addition, the book will also contain some practical applications of these problems in systems biology, etc. In order to study different types of systems, various methods, e.g., a

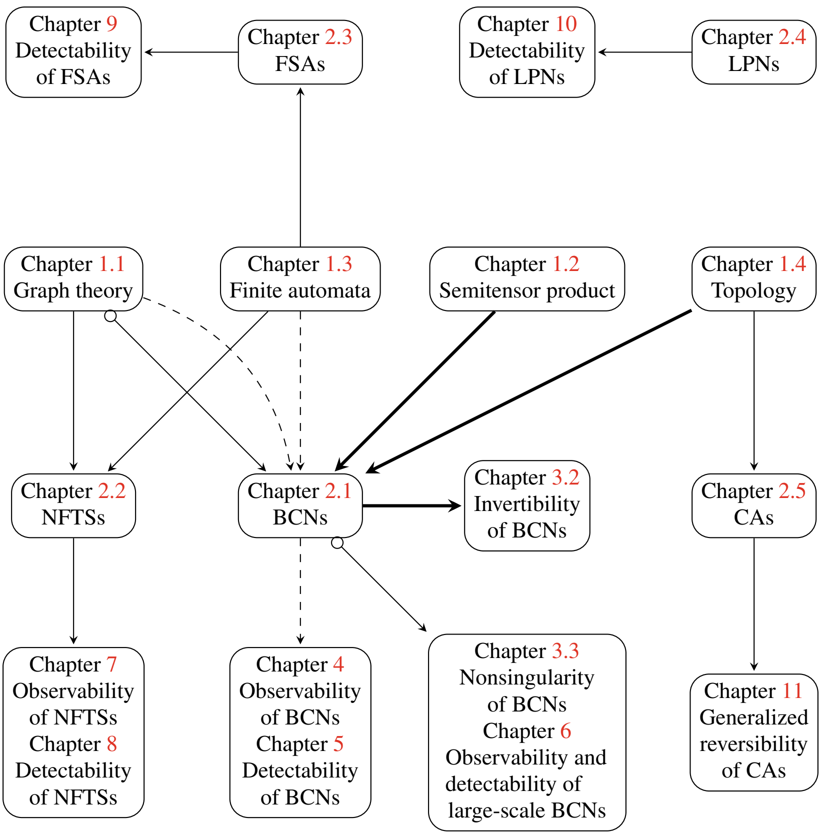

Fig. 0.1 Reading flow of the book

semitensor product method, a graph-theoretic method, a finite-automaton method, a topological method, etc., will be adopted. The book is aiming at bringing the reader new understanding of discrete-time and discrete-space dynamical systems.

While reading the book, the reader could refer to the reading flow shown in Fig. 0.1 through arrow lines of the same type. We did not introduce the mathematical tools used to handle labeled Petri nets in Chap 1, but introduced them in Chap. 10 when labeled Petri nets were studied, because such tools quite depend on the labeled Petri nets themselves.

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Contents

Abbreviations

$\emptyset$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{Z}$	Set of integers
$\mathbb{Z}_+$	Set of positive integers
$\mathbb{R}$	Set of real numbers
$\mathbb{R}_{\geq 0}$	Set of nonnegative real numbers
$\mathbb{R}^n$	Set of n-dimensional real column vectors
$\mathbb{R}^{m\times n}$	Set of $m \times n$ real matrices
$\mathcal{L}_{m\times n}$	Set of $m \times n$ logical matrices
$\lvert E\rvert$	Cardinality of set E
2^E	Power set of set E
Id	Identity map
$\mathrm{ord}_x(\alpha)$	Order of α at x
$\mathrm{ord}(\alpha)$	Order of α
$\beta \succ \alpha$	β is finer than α
$D(\alpha)$	Minimum of $\mathrm{ord}(\beta)$ for all $\beta \succ \alpha$
$\dim(X)$	Covering dimension of topological space X
$A \cap B$	Intersection of sets A and B
$A \cup B$	Union of sets A and B
$A \uplus B$	Disjoint union of sets A and B
$A \backslash B$	Difference of sets A and B
$A \subset B$	Set A is a subset of set B
$B_r(x)$	Ball centered at x with radius r
$\sup E$	Supremum of set E
$\inf E$	Infimum of set E
$\mathring{E}$ or $\mathrm{Int}(E)$	Interior of set E
$\overline{E}$ or $\mathrm{Clo}(E)$	Closure of set E
$\mathrm{diam}(E)$	Diameter of set E
$\mathrm{dist}(x, E)$	Distance of point x to set E
$[a, b]$	Closed interval with endpoints a and b ($a \leq b$)

$[a;b]$	Set of integers no less than a and no greater than b ($a \leq b$)
(a,b)	Open interval with endpoints a and b ($a \leq b$)
$[a,b)$	$[a,b]\setminus\{b\}$
$(a,b]$	$[a,b]\setminus\{a\}$
S^*	Set of words over alphabet S
S^+	Set of words over alphabet S excluding the empty word ϵ
S^ω	Set of configurations over alphabet S
$[u]$	Cylinder determined by pattern u
$S^{\mathbb{Z}^d}$	Symbolic space over alphabet S
$\mathcal{D}$	$\{0,1\}$
$\mathrm{lcm}(p,q)$	Least common multiple of positive integers p and q
$\gcd(p,q)$	Greatest common divisor of positive integers p and q
I_n	Identity matrix of order n
δ_n^i	i-th column of the identity matrix I_n
$\mathbf{1}_n$	$\sum_{i=1}^n \delta_n^i$
Δ_n ($\Delta_2 =: \Delta$)	Set of the columns of the identity matrix I_n
A^T	Transpose of matrix A
$\mathrm{Col}_i(A)$	i-th column of matrix A
$\mathrm{Col}(A)$	Set of columns of matrix A
$\mathrm{Row}_i(A)$	i-th row of matrix A
$\mathrm{Row}(A)$	Set of rows of matrix A
$\ltimes$	Semitensor product
$\otimes$	Kronecker product
$A_1 \oplus \cdots \oplus A_n$	$\begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_n \end{bmatrix}$