Geometry of Surfaces

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A Practical Guide for Mechanical Engineers

Second Edition

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This book is dedicated to my wife Natasha

Preface

This book is about geometry of part surfaces, their generation, and interaction with one another.

Written by a mechanical engineer, this book is ***not*** on differential geometry of surfaces. Instead, the book is devoted to the application of methods those developed in differential geometry of surfaces, for the purposes of solving problems in mechanical engineering.

A paradox exists in our current understanding of geometry of surfaces: We know everything about ideal (perfect) surfaces, which do not exist in reality, and our knowledge about real surfaces that exist physically is limited. Therefore, one of the main goals of the book is to adjust our knowledge of ideal surfaces for the purposes of better understanding of geometry of real surfaces. In other words: the goal of the book is to bridge a gap between ideal and real surfaces.

One of the significant advantages of the book is due to that it is written ***not*** by a mathematician, but it is written by a mechanical engineer, and for mechanical engineers.

Southfield, MI, USA Stephen P. Radzevich

Acknowledgements

I would like to share the credit for any research success with my numerous doctoral students with whom I have tested the proposed ideas and applied them in the industry. The contributions of many friends, colleagues, and students in overwhelming number cannot be acknowledged individually, and as much as our benefactors have contributed, even though their kindness and help must go unrecorded.

My thanks also go to those at *Springer* who took over the final stages and who will have to cope with the marketing and sales of the fruit of my efforts.

Introduction

Performance of parts depends in much on geometry of the interacting surfaces. An in-depth investigation of geometry of smooth regular part surfaces is undertaken in this book. Analytical description of the surfaces, methods of their generation, along with an analytical approach for description of the geometry of contact of the interacting part surfaces are covered in the book.

This book is comprised of three parts and appendices.

Specification of part surfaces in terms of corresponding nominal smooth regular surface is considered in Part I of the book.

Geometry of part surfaces is discussed in Chap. 1 of the book. The discussion begins with an analytical description of perfect (ideal) surfaces. Here, in this text ideal surface is interpreted as a zero-thickness film. Then difference between *Classical Differential Geometry* and *Engineering Geometry of Surfaces* is analyzed. This analysis is followed by an analytical description of real part surfaces, which is based on much of the analytical description of the corresponding ideal surface. It is shown that while remained unknown, real part surface is located in between two boundary surfaces. The said boundary surfaces are represented by two perfect surfaces of the upper tolerance and of the lower tolerance. Specification of surfaces ends with the discussion of the natural representation of a desired part surface. This consideration involves the first and the second fundamental forms of a smooth regular part surface. For an analytically specified surface, elements of its local geometry are outlined. This consideration includes but is not limited to analytical representation of unit tangent vectors, tangent plane, unit normal vector, unit vectors of principal directions on a part surface, and so forth. Ultimately, design parameters of part surface curvature are discussed. Mostly, equations for principal surface curvatures along with normal curvatures at a surface point are considered. In addition to that mean curvature, *Gaussian* curvature, absolute curvature, shape operator, and curvedness of a surface at a point are considered. The classification of local part surface patches is proposed in this section of the book. The classification is followed by a circular chart comprised of all possible kinds of local part surface patches.

Chapter 2 is devoted to the analysis of a possibility of classification of part surfaces. Regardless of no scientific classification of smooth regular surfaces in global sense is feasible in nature, local part surface patches can be classified. For the investigation of geometry of local part surface patches, planar characteristic images are employed. In this analysis, *Dupin* indicatrix, curvature indicatrix, and circular diagrams at a part surface point are covered in detail. Based on the obtained results of the analysis, two more circular charts are developed. One of them employs the part surface curvature indicatrices, while another one is based on the properties of circular diagrams at a current part surface point. This section of the book ends with a brief consideration of one more useful characteristic curve, which can be helpful for analytical description of geometry of a part surface locally.

In Part II, geometry of contact of two smooth regular part surfaces is considered. This part of the book is comprised of four chapters.

In Chap. 3, the discussion begins with review of earlier works in the field of contact geometry of surfaces. This includes order of contact of two surfaces, local relative orientation of the surfaces at a point of their contact, the first-order and the second-order analysis. The first-order analysis is limited just to the common tangent plane. The second-order analysis begins with the author's comments on analytical description of the local geometry of contacting surfaces loaded by a normal force: *Hertz*'s proportional assumption. Then, the surface of relative normal curvature is considered. *Dupin* indicatrix and curvature indicatrix of the surface of relative normal curvature are discussed. This analysis is followed by the discussion of the surface of relative normal radii of curvature, normalized relative normal curvature along with a characteristic curve $Ir_k(\mathscr{R})$ of novel kind.

This section of the monograph is followed by Chap. 4, in which an analytical method based on second fundamental forms of the contacting part surfaces is discussed. It is shown there that the resultant deviation of one of the contacting surfaces from the other contacting surface expressed in terms of the second fundamental forms of the contacting surfaces could be the best possible criterion for the analytical description of the contact geometry of two smooth regular surfaces. Such a criterion is legitimate, but it is computationally impractical. Thus, other analytical methods are required to be developed for this purpose.

In Chap. 5, a novel kind of characteristic curve for the purposes of analytical description of the contact geometry of two smooth regular part surfaces in the first order of tangency is discussed in detail. The discussion begins with preliminary remarks and follows with introduction and with derivation of an equation of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two part surfaces. Then, directions of extremum degree of conformity of two part surfaces in contact are specified and are described analytically. This analysis is followed by determination and by derivation of corresponding equations of asymptotes of the indicatrix of conformity $Cnf_R(P_1/P_2)$. Capabilities of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two smooth regular part surfaces P_1 and P_2 in the first order of tangency are compared with the corresponding capabilities of "*Dupin indicatrix Dup* ($\mathscr{R}$)" of the surface of relative curvature $\mathscr{R}$. Important properties of the indicatrix of conformity of two smooth regular part surfaces are outlined. Ultimately, the converse indicatrix of conformity $Cnf_R^{cnv}(P_1/P_2)$ of two regular part surfaces in the first order of tangency is introduced and is briefly discussed as an alternative to the regular indicatrix of conformity $Cnf_R(P_1/P_2)$.

In Chap. 6, more characteristic curves are derived on the premises of "*Plücker conoid*" constructed at a point of a smooth regular part surface. At the beginning, main properties of the surface of "*Plücker conoid*" are briefly outlined. This includes but not limited to basics, analytical representation, and local properties along with auxiliary formulae. This analysis is followed by analytical description of local geometry of a smooth regular part surface. Ultimately, expressions for two more characteristic curves are derived. These newly introduced characteristic curves are referred to as *Plücker* curvature indicatrix and $\mathscr{P}lk_R(P_1)$-indicatrix of a part surface. The performed analysis makes it possible derivation of equations for two more planar characteristic curves for analytical description of the contact geometry of two smooth regular part surfaces P_1 and P_2 at a point of their contact. One of the newly derived characteristic curves is referred to as "$\mathscr{P}lk_R(P_1/P_2)$*-relative indicatrix of the first kind*" of two contacting part surfaces P_1 and P_2. Another one in a curve inverse to the characteristic curve $\mathscr{P}lk_R(P_1/P_2)$. This second characteristic curve is referred to as "$\mathscr{P}lk_k(P_1/P_2)$*-relative indicatrix of the second kind*." Main properties of both the characteristic curves are briefly discussed in this section of the monograph.

Feasible kinds of contact of two smooth regular part surfaces in the first order of tangency are discussed in Chap. 7. Analytical description of contact geometry of two smooth regular surfaces begins with investigation of a possibility of implementation of the indicatrix of conformity for the purposes if identification of actual kind of contact of two smooth regular part surfaces. Then, impact of accuracy of the computation of the parameters of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two part surfaces is investigated. Ultimately, a classification of all possible kinds of contact of two smooth regular part surfaces in the first order of tangency is developed.

Various kinds of mapping of a part surface onto another part surface are discussed in Part III of the monograph. The discussion in this part of the monograph begins with a novel kind of the surfaces mapping, the so-called $\mathbb{R}$-mapping of the interacting part surfaces.

In Chap. 8, a novel method of the surfaces mapping, namely $\mathbb{R}$-mapping of the interacting part surfaces is disclosed. The preliminary remarks on the developed approach is followed by an in-detail consideration of the concept underlying in the $\mathbb{R}$-mapping of the interacting part surfaces. Then, principal features of $\mathbb{R}$-mapping of a part surface P_1 onto another part surface P_2 are disclosed. Due to $\mathbb{R}$-mapping of surface returns an equation of the mapped surface in natural representation, namely, in terms of fundamental magnitudes of the first and of the second order, the derived equation of the mapped surface is required been reconstructed and been represented in a convenient reference system. This issue got a comprehensive discussion in this chapter of the monograph. Consideration in the chapter ends with two examples of implementation of the discussed method of part surfaces mapping.

General consideration of generation of enveloping surface is discussed in Chap. 9. The consideration begins with the analysis of generation of an envelope to successive positions of a moving planar curve. Then, the discussion is extended to generation of the enveloping surface to successive positions of a moving smooth regular part surface. Enveloping surfaces to one-parametric as well as to two-parametric family of surfaces are covered in this section of the monograph. Further, the "*kinematic* *method*" for generation of enveloping surfaces is introduced. The method was developed in 1940 by Dr. V. A. Shishkov. Implementation of the kinematic method for generation of one-parametric enveloping surfaces is disclosed. Then, the approach is extended to multi-parametric motion of a smooth regular part surface.

In Chap. 10, special cases of generation of enveloping surfaces are disclosed. For this purpose, a concept of reversibly enveloping surfaces is introduced. For the generation of reversibly enveloping surfaces, a novel method is proposed. This method is illustrated by an example of generation of reversibly enveloping surfaces in case tooth flanks for geometrically accurate (ideal) crossed-axis gear pairs. The performed analysis makes it possible a conclusion that two *Olivier principles* of generation of enveloping surfaces:

- in general case are not valid and
- in a degenerate case these two principles are useless

Ultimately, there is no sense to apply *Olivier principles* for the purpose of generation of reversibly enveloping smooth regular part surfaces.

Part surfaces those allow for sliding over themselves are considered as a particular degenerated case of enveloping surfaces.

Appendices' section contains reference material that is useful in practical applications.

Elements of vector algebra are briefly outlined in Appendix A.

In Appendix B, elements of coordinate system transformation are represented. This section of the book also includes direct transformation of the surface fundamental forms. The latter makes it possible to avoid calculation of the first and of the second derivatives of the part surface equation, after the equation is represented in a new reference system.

Formulae for changing of surface parameters are represented in Appendix C.

In Appendix D, the closest distance of approach between two smooth regular surfaces is discussed.

A book of this size is likely to contain omissions and errors. If you have any constructive suggestions, please communicate them to the author via e-mail: sp_radzevich@yahoo.com.

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Contents

About the Author

Dr. Stephen P. Radzevich is Professor of Mechanical Engineering and Professor of Manufacturing Engineering. He received the M.Sc. (1976), the Ph.D. (1982), and the Dr.(Eng)Sc. (1991) all in mechanical engineering. He has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with *CAD* and *CAM* of precise gear finishing for a variety of industrial sponsors. His main research interest is *kinematic geometry of surface generation*, particularly with the focus on (a) precision gear design, (b) high-power-density gear trains, (c) torque share in multi-flow gear trains, (d) design of special purpose gear cutting/finishing tools, (e) design and machining (finishing) of precision gears for low-noise/noiseless transmissions of cars, light trucks, *etc*. He has spent about 40 years developing software, hardware, and other processes for gear design an optimization. Besides his work for industry, he trains engineering students at universities and gear engineers in companies. He authored and co-authored over 30 monographs, handbooks, and textbooks. The monographs entitled *Generation of Surfaces* (2001), *Kinematic Geometry of Surface Machining* (*CRC* Press, 2008), *CAD/CAM of Sculptured Surfaces on Multi-Axis NC Machine: The DG/K-Based Approach* (M&C Publishers, 2008), *Gear Cutting Tools: Fundamentals of Design and Computation* (*CRC* Press, 2010, 2nd edition 2017), *Precision Gear Shaving* (*Nova Science Publishers*, 2010), *Dudley's Handbook of Practical Gear Design and Manufacture* (*CRC* Press, 2012, 2nd edition 2016), *Theory of Gearing: Kinematics,* *Geometry, and Synthesis* (*CRC* Press, 2012, 2nd edition 2018), *Geometry of Surfaces: A Practical Guide for Mechanical Engineers* (2013) are among the recently published volumes. He also authored and co-authored about 350 scientific papers and holds over 260 patents on inventions in the field of his scientific interests, both, USA patents on inventions, and International.

Notations

A_{P1}	Is the apex of the base cone of the part surface P_1
A_{P2}	Is the apex of the base cone of the part surface P_2
A_{pa}	Is the plane-of-action apex
C	Is the center distance
$C_{1.P1}$, $C_{2.P1}$	Are the first and second principal plane sections of the traveling part surface P_1
$C_{1.P2}$, $C_{2.P2}$	Are the first and second principal plane sections of the generated part surface P_2 (the enveloping surface)
$Cnf_R(P_1/P_2)$	Is the indicatrix of conformity at a point of contact of two smooth regular part surfaces P_1 and P_2
$Cnf_k(P_1/R_2)$	Is the indicatrix of conformity that is converse to the indicatrix $Cnf_R\,(P_1/P_2)$
E	Is the characteristic line
E_{P1}, F_{P1}, G_{P1}	Are the fundamental magnitudes of the first order of the smooth regular part surface P_1
E_{P2}, F_{P2}, G_{P2}	Are the fundamental magnitudes of the first order of the smooth regular part surface P_2
G_P	Is the full curvature of a part surface P at a point m
M_P	Is the mean curvature of a surface P at a point m
K	Is the point of contact of two smooth regular part surfaces P_1 and P_2 (or a point within a line of contact of the part surfaces P_1 and P_2)
LC	Is the line of contact between two regular part surfaces P_1 and P_2
L_{P1}, M_{P1}, N_{P1}	Are the fundamental magnitudes of the second order of the smooth regular part surface P_1
L_{P2}, M_{P2}, N_{P2}	Are the fundamental magnitudes of the second order of the smooth regular part surface P_2
O_{P1}	Is the axis of rotation of the part surface P_1
O_{P2}	Is the axis of rotation of the part surface P_2
O_{pa}	Is the axis of rotation of the plane of action, *PA*

PA	Is the plane of action
P_{ln}	Is the axis of instant rotation of two regular part surfaces P_1 and P_2 in relation to one another
$\mathbf{Rc}\,(PA \mapsto G)$	Is the operator of rolling/sliding (the operator of transition from the plane of action, PA, to the gear, G, in crossed-axes gearing)
$\mathbf{Rc}\,(PA \mapsto P)$	Is the operator of rolling/sliding (the operator of transition from the plane of action, PA, to the pinion, P, in crossed-axes gearing)
$\mathbf{Rl}_x(\varphi_y,\ Y)$	Is the operator of rolling over a plane (Y-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_y,\ Y)$	Is the operator of rolling over a plane (Y-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_x,\ X)$	Is the operator of rolling over a plane (X-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_x,\ X)$	Is the operator of rolling over a plane (X-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_x(\varphi_z,\ Z)$	Is the operator of rolling over a plane (Z-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_z,\ Z)$	Is the operator of rolling over a plane (Z-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rr}_u(\varphi,\ Z)$	Is the operator of rolling of two coordinate systems
$\mathbf{Rs}\,(A \mapsto B)$	Is the operator of the resultant coordinate system transformation, say from a coordinate system A to a coordinate system B
$\mathbf{Rt}\,(\varphi_x,\ X)$	Is the operator of rotation through an angle φ_x about X-axis
$\mathbf{Rt}\,(\varphi_y,\ Y)$	Is the operator of rotation through an angle φ_y about Y-axis
$\mathbf{Rt}\,(\varphi_z,\ Z)$	Is the operator of rotation through an angle φ_z about Z-axis
$R_{1.P1},\ R_{2.P1}$	Are the first and the second principal radii of the gear tooth flank P_1
$R_{1.P2},\ R_{2.P2}$	Are the first and the second principal radii of the gear tooth flank P_2
$\mathbf{Sc}_x(\varphi_x,\ p_x)$	Is the operator of screw motion about the X-axis
$\mathbf{Sc}_y(\varphi_y,\ p_y)$	Is the operator of screw motion about the Y-axis
$\mathbf{Sc}_z(\varphi_z,\ p_z)$	Is the operator of screw motion about the Z-axis
$\mathbf{Tr}\,(a_x,\ X)$	Is the operator of translation at a distance a_x along the X-axis
$\mathbf{Tr}\,(a_y,\ Y)$	Is the operator of translation at a distance a_y along the Y-axis
$\mathbf{Tr}\,(a_z,\ Z)$	Is the operator of translation at a distance a_z along the Z-axis
$U_{P1},\ V_{P1}$	Is the curvilinear (gaussian) coordinates of a point of a smooth regular part surface P_1
$U_{P2},\ V_{P2}$	Is the curvilinear (gaussian) coordinates of a point of a smooth regular part surface P_2
$\mathbf{U}_{P1},\ \mathbf{V}_{P1}$	Are the tangent vectors to curvilinear coordinate lines on a smooth regular part surface P_1
$\mathbf{U}_{P2},\ \mathbf{V}_{P2}$	Are the tangent vectors to curvilinear coordinate lines on a smooth regular part surface P_2

$\mathbf{V}_{\Sigma}$	Is the linear velocity vector of the resultant motion of the smooth regular part surface P_1 in relation to a reference system, which the smooth regular part surface P_2 will be associated with
$d_{\mathrm{cnf}}^{\min}$	Is the minimal diameter of the indicatrix of conformity, $Cnf_R(P_1/P_2)$, for two smooth regular part surfaces P_1 and P_2 at a current contact point K
$k_{1.P1}$, $k_{2.P1}$	Are the first and second principal curvatures at a point of the smooth regular part surface P_1
$k_{1.P2}$, $k_{2.P2}$	Are the first and second principal curvatures at a point of the smooth regular part surface P_2
$\mathbf{n}_P$	Is the unit normal vector to a smooth regular part surface P
p_{sc}	Is the screw parameter (reduced pitch) of instant screw motion of the part surface P_1in relation to the part surface P_2
$\mathbf{r}_{P1}$	Is the position vector of a point of a smooth regular part surface P_1
r_{cnf}	Is the position vector of a point of the indicatrix of conformity $Cnf_R(P_1/P_2)$ at a point of contact of two smooth regular part surfaces P_1 and P_2
$\mathbf{t}_{1.P1}$, $\mathbf{t}_{2.P1}$	Are the unit tangent vectors of principal directions at a point of the smooth regular part surface P_1
$\mathbf{t}_{1.P2}$, $\mathbf{t}_{2.P2}$	Are the unit tangent vectors of principal directions at a point of the smooth regular part surface P_2
$\mathbf{u}_{P1}$, $\mathbf{v}_{P1}$	Are the unit tangent vectors to curvilinear coordinate lines at a point of the smooth regular part surface P_1
$\mathbf{u}_{P2}$, $\mathbf{v}_{P2}$	Are the unit tangent vectors to curvilinear coordinate lines at a point of the smooth regular part surface P_2
$x_P y_P z_P$	Is the local *cartesian* coordinate system having its origin at a current point of contact of the part surfaces P_1 and P_2

Greek Symbols

$\Phi_{1.P1}$, $\Phi_{2.P1}$	Are the first and second fundamental forms of the smooth regular part surface P_1
$\Phi_{1.P2}$, $\Phi_{2.P2}$	Are the first and second fundamental forms of the smooth regular part surface P_2
$\phi_{t.\omega}$	Is the transverse pressure angle
μ	Is the angle of the part surfaces P_1 and P_2 local relative orientation
$\boldsymbol{\omega}_{P1}$	Is the rotation vector of the regular part surface P_1
$\boldsymbol{\omega}_{P2}$	Is the rotation vector of the part surface P_2
$\boldsymbol{\omega}_{pl}$	Is the vector of instant rotation of the part surfaces P_1 and P_2 in relation to one another

Subscripts

cnf	Conformity
max	Maximum
min	Minimum
t	Tangential
opt	Optimal