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Analysis and Design of Delayed Genetic Regulatory Networks

 Springer

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To Chunyan and Ruitong

Xian Zhang

To My Family

Yantao Wang

To My Family

Ligang Wu

Preface

The research on genetic regulatory networks (GRNs) is multi-disciplinary, crossing biology, control science, computer science, electronic science mathematics, theoretical physics, etc. In the last two decades, mathematical models have become a powerful tool for studying of GRNs. In general, mathematical models of GRNs are divided into two classes: the discrete models and the continuous ones. In the continuous models, the variables describe the concentrations of mRNAs and proteins as continuous values, which can provide detailed understanding of the non-linear dynamical behavior exhibited by GRNs. Recently, it has been shown that differential equation models including delayed states, named as delayed GRNs, can more accurately describe GRNs. As a result, much effort has been paid to the study of delayed GRNs, and many significant results have been reported in literature. From the point of control theory, the research on delayed GRNs includes mainly two aspects: analysis and design. In spite of the fact that there exist some Ph.D. theses related to delayed GRNs, there is no comprehensive book on this topic.

The aim of this book is to provide an introduction for current advances of delayed GRNs and present the basic methods for analysis and design of delayed GRNs. The whole book is divided into 11 chapters and focuses on the analysis and design problems on continuous-time delayed GRNs except the last chapter. All the contexts are taken from the authors' publications. This book is also intended to offer a collection of important references on analysis and design of delayed GRNs.

The book is addressed to graduate students and research-level mathematicians. It is hoped that the book will be suitable for postgraduate use or as a reference.

Many researchers in the world have made great contribution to analysis and design of delayed GRNs. Due to the length limitation and the structural arrangement, many of their published results are not included in the book. I would extend my apologies to these researchers.

I would appreciate any comments and corrections from the readers. Please feel free to contact me by the e-mail: xianzhang@hlju.edu.cn.

Harbin, China
February 2019

Xian Zhang

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Notations and Acronyms

R	Field of real numbers
$\mathbf{R}^{n \times m}$	Set of all $n \times m$ matrices over R
\mathbf{R}^n	Set $\mathbf{R}^{n \times 1}$
$\text{diag}(x_1, \dots, x_n)$ or D_x	Diagonal matrix
$\text{col}(x_1, \dots, x_n)$	Column vector
$\lambda_{\max}(A)$	Maximum eigenvalue of real symmetric matrix A
$\lambda_{\min}(A)$	Minimum eigenvalue of real symmetric matrix A
$\chi_j(A)$	Number of nonzero elements in the j th row of A
$\chi(A)$	$\text{diag}(\chi_1(A), \chi_2(A), \dots, \chi_n(A))$
$ A $	Matrix $[a_{ij}]$ with $A = [a_{ij}]$
\prod	Product sign
\sum	Sum sign
I_n or I	$n \times n$ identity matrix
$0_{m \times n}$ or 0	$m \times n$ zero matrix
A^T	Transpose of the matrix A
A^*	Conjugate transpose of the matrix A
$\det(A)$	Determinant of the square matrix A
$\text{tr}(A)$	Trace of the square matrix A
$\rho(A)$	Spectral radius of the square matrix A
$\text{sym}(A)$	Matrix $A + A^T$
A^{-1}	Inverse of the nonsingular matrix A
$\langle m \rangle$	Set $\{1, 2, \dots, m\}$
\circ	Hadamard produce
$X \geq Y$ or $Y \leq X$	$X - Y$ is real symmetric positive semi-definite
$X > Y$ or $Y < X$	$X - Y$ is real symmetric positive definite
$L_2[0, \infty)$	Set of square integrable functions over $[0, \infty)$
J	Connected subset of R
$C(\mathbf{J}, \mathbf{R}^n)$	Linear space of all continuous functions $h : \mathbf{J} \rightarrow \mathbf{R}^n$
$\ \cdot\ _2$	Euclidean norm on \mathbf{R}^n , or its induced norm

$\mathcal{C}((-\infty, 0], \mathbf{R}^n)$	Linear space of all bounded and uniformly continuous functions $\psi : (-\infty, 0] \rightarrow \mathbf{R}^n$
$\ \psi\ _C$	Norm on $\mathcal{C}((-\infty, 0], \mathbf{R}^n)$ defined by $\ \psi\ _C = \sup_{-\infty < s \leq 0} \ \psi(s)\ _2 + \int_{-\infty}^0 \ \psi(s)\ _2 ds$
\mathcal{R}	Compact set in \mathbf{R}^l
$\partial\mathcal{R}$	Boundary of \mathcal{R}
$C^1(\mathcal{R}, \mathbf{R}^n)$	Banach space of continuous differential functions mapping \mathcal{R} into \mathbf{R}^n
$\ \cdot\ $	Norm on $C^1(\mathcal{R}, \mathbf{R}^n)$ defined by $\ y(x)\ = (\int_{\mathcal{R}} y^T(x)y(x)dx)^{1/2}$
$\ \cdot\ _d$	Norm on $C^1([-d, 0] \times \mathcal{R}, \mathbf{R}^n)$ defined by $\ \phi(t, x)\ _d = \max \left\{ \sup_{-d \leq t \leq 0} \ \phi(t, x)\ , \sup_{-d \leq t \leq 0} \left\ \frac{\partial \phi(t, x)}{\partial t} \right\ , \max_{1 \leq k \leq l} \sup_{-d \leq t \leq 0} \left\ \frac{\partial \phi(t, x)}{\partial x_k} \right\ \right\}$
$\mathbf{E}\{\cdot\}$	Mathematical expectation operator
\mathcal{L}	Weak infinitesimal operator
$A \preceq B$ or $B \succeq A$	Real matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ satisfy $a_{ij} \leq b_{ij}$ for all i and j
$A \prec B$ or $B \succ A$	Real matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ satisfy $a_{ij} < b_{ij}$ for all i and j

Next, we will briefly explain several phrases which are helpful to understand this book.

- The LKF is a class of nonnegative functionals acting on a space of functions.
- Jensen’s inequality relaxes the integral term of quadratic quantities into the quadratic term of the integral quantities and results in a linear combination of positive functions weighted by the inverses of convex parameters.
- The free-weighting matrix approach introduces parameter matrices, indicating the relationship between the terms in the Leibniz–Newton formula, into LMIs to be solved.
- The convex technique simplifies LMIs, including a linear combination of finite items weighted by convex parameters, by using the property of convex functions.
- The reciprocally convex technique estimates the lower boundary of a combination of positive functions weighted by the inverses of convex parameters.
- The Wirtinger-type integral inequality is a class of inequalities that are a generalization of Jensen’s inequalities and that are more accurate than Jensen’s inequalities.
- A delay-dependent(-independent) result indicates solvable conditions to a problem on time-delay systems are (not) related to the information of delay.

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