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Luís Barreira · Claudia Valls

Dynamical Systems by Example

Springer

Luís Barreira
Instituto Superior Técnico
Universidade de Lisboa
Lisbon, Portugal

Claudia Valls
Instituto Superior Técnico
Universidade de Lisboa
Lisbon, Portugal

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Preface

This book is a large collection of problems, all with detailed solutions, on the core of the theory of dynamical systems. Besides the basic theory, the topics include topological dynamics, low-dimensional dynamics, hyperbolic dynamics, symbolic dynamics, and basic ergodic theory.

As in any other area of mathematics, it is important while learning dynamical systems to solve selected problems (always after a careful study of the material!), in particular to get a first working knowledge of the topics as well as of some less direct difficulties. It goes without saying that this helped substantially generations of mathematicians in getting a solid understanding of the theory and of its applications. Nevertheless, it is difficult to find large collections of problems on less basic subjects, at least other than on quite specific topics. Moreover, these problems often lack detailed solutions or even any solutions, while it would certainly be quite helpful for a student to have the possibility to study detailed solutions, especially if studying independently.

Certainly, it is not a good practice to study detailed solutions of exercises without first trying hard to solve them. Indeed, solving exercises is an important step toward learning a particular subject and ultimately toward becoming an independent mathematician, while perhaps also developing a personal style. On the other hand, it would be quite welcome to have available comprehensive sources of problems worked out in detail. In more advanced topics, these solutions can have the role of providing detailed arguments for comparison or even alternative views, possibly alerting to more direct approaches or to less obvious connections to other topics (and mathematics is full of such connections).

Our text is a contribution to fill this gap on selected topics of the theory of dynamical systems (as detailed below). It can be used as a companion to a textbook for a one-semester or two-semester course on dynamical systems at the advanced undergraduate or beginning graduate levels, or for independent study of those topics. Other than some basic pre-requisites from linear algebra, differential and integral calculus, complex analysis and topology, in each chapter we recall all notions and results (without proofs) that are necessary for the problems in that chapter, thus making the text self-contained.

The theory of dynamical systems is quite broad and active in terms of research. Hence, it was necessary to make a careful selection of the material. In this aspect, we followed closely our book [11], which gives an introduction to the theory of dynamical systems and to which the present book can be an excellent companion. The levels of the two texts are analogous, although the present book provides a more comprehensive working knowledge of the topics as well as a much larger variety of problems, also of various levels of difficulty (from simple to quite elaborate) and with a carefully planned interdependence when appropriate, so that the material is learned in successive steps.

We detail briefly the topics addressed in the book together with recommendations for further reading:

- Chapter I.1 considers the notion of a dynamical system, both for discrete and continuous time, invariant sets, orbits, periodic points, rotations and expanding maps of the circle, endomorphisms and automorphisms of the torus, autonomous differential equations and their flows, Poincaré sections, and Poincaré maps (see [3, 10, 20, 22]).
- Chapter I.2 studies continuous maps of a topological space and their topological properties, including the notions of α-limit set and of ω-limit set, recurrent points, nonwandering points, minimal sets, topological transitivity, topological mixing, and topological conjugacies, as well as topological entropy and its properties (see [16, 17, 48]).
- Chapter I.3 considers dynamical systems on low-dimensional spaces, including homeomorphisms of the circle, their lifts, orientation-preserving homeomorphisms and the notion of rotation number, continuous maps on the interval, and the Poincaré–Bendixson theory on the plane (see [2, 10, 22, 24]).
- Chapter I.4 studies hyperbolic dynamics, including the notion of a hyperbolic set, the Smale horseshoe, invariant families of cones, topological conjugacies, and invariant manifolds near a hyperbolic fixed point, as well as geodesic flows and their hyperbolicity (see [28, 40, 41, 50, 54]).
- Chapter I.5 considers selected topics of symbolic dynamics, including one-sided and two-sided shift maps, topological Markov chains, irreducible and transitive matrices, topological transitivity, topological mixing, topological entropy, and zeta functions (see [16, 33, 34]).
- Chapter I.6 studies selected basic topics of ergodic theory, including Poincaré's recurrence theorem, Birkhoff's ergodic theorem, and the notion of metric entropy (see [8, 23, 35, 46, 56]).

The book is conveniently separated into two parts. Part I (Chaps. I.1–I.6) recalls briefly in each chapter various notions and results that are needed for the problems of that chapter, also with the purpose of fixing notation, after which the problems are formulated without their solutions. In Part II (Chaps. II.1–II.6) the problems are restated but now with their solutions. In this way each problem can be solved without the risk of looking inadvertently at the solution. There are 240 problems ranging from simple to quite elaborate (with 40 per chapter), all with detailed solutions, on the basic theory, topological dynamics, low-dimensional dynamics, hyperbolic dynamics, symbolic dynamics, and basic ergodic theory. The text is complemented by 50 figures.

We also provide a list of important topics, certainly incomplete, that were left out of the book for reasons of scope together with recommendations for further reading:

- Holomorphic dynamics (see [13, 18, 36, 37, 38, 49]);
- Bifurcation theory and normal forms (see [5, 10, 21, 25]);
- Hamiltonian dynamics (see [1, 6, 10, 29, 39]);
- Discrete groups of isometries (see [4, 12, 31]);
- Dimension theory and multifractal analysis (see [7, 44]);
- Thermodynamic formalism and its applications (see [15, 32, 51]);
- Hyperbolicity and homoclinic bifurcations (see [42]);
- Hyperbolic dynamics and zeta functions (see [43]);
- Partial hyperbolicity and stable ergodicity (see [45]);
- Nonuniform hyperbolicity and smooth ergodic theory (see [9, 14, 47]);
- Hyperbolic systems with singularities and billiards (see [19, 30]);
- Algebraic dynamics and ergodic theory (see [52]);
- Infinite-dimensional dynamics (see [26, 27, 53, 55]).

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Luís Barreira
Claudia Valls

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