

Advanced Vehicle Dynamics

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*We live twice.
The first round is serious.
The second round is funny.
Interestingly, we are mixed together on Earth.*

Dedicated to
Kavosh,
Vazan,
Mojgan.

Preface

In this book, we revise the vehicle dynamics based on a new mathematical model for combined tire forces. The force system at the tireprint of a loaded, rolling, steered, cambered tire includes forward, lateral, and vertical forces, as well as aligning, roll, and pitch moment. The forward and lateral forces are the most significant forces in vehicle maneuvering. Tire force modeling has been introduced more than a century ago and improved in several steps through experiments and empirical modeling.

In dynamic modeling of tire forces and vehicles motion, it is traditional to introduce the longitudinal force by slip ratio and lateral force by sideslip angle. Our experiments as well as other available experiments conducted by other investigators during the past decades show that there are decreasing interactions between the lateral and longitudinal tire forces. We introduce a set of equations to model interaction of the lateral and longitudinal tire forces and develop the equations of motion of vehicles based on a new model. Such realistic tire force model has the potential to improve the control strategies and increase the safety of vehicles at critical conditions.

In summary, the vehicle dynamic models that are developed for combined tire forces modeling, four-wheel planar, bicycle roll, and four-wheel roll are new theories introduced in this book. Several examples have been included to show the effectiveness of the mathematical equations as well as good results to compare other analysis and projects. The newly developed equations of motion and the mathematical modeling are perfect for investigation, study, prediction, and development of control strategy specially for vehicle drift, sliding, and skidding on slippery, icy, snowy, wet, dirt pavements. The equations are also supporting the new vehicle designs equipped with in-wheel electrical motors, and steering by wires, as well as traditional vehicle designs.

I deeply appreciate the extensive helps from my colleagues, Hormoz Marzbani, Sina Milani, Nguyen Dang Quy, and Amir Salemi, for their valuable comments and reviews, simulation, and tests. This book would have not been prepared without their contributions.

Level of the Book

This book has been developed from nearly a decade of research and experiments in nonlinear vehicle dynamics and teaching courses in vehicle dynamics. It is addressed primarily to the graduate student in engineering. Hence, it is an advanced releaser book that may also be used as a textbook. It provides fundamental and advanced topics needed in computerizing vehicle handling. The whole book can be covered in one course in 12–16 weeks. Students are required to know the fundamentals of vehicle dynamics, kinematics and dynamics, as well as have an acceptable knowledge of numerical methods and differential equations.

The contents of the book have been kept at a fairly theoretical–practical level. All concepts are deeply explained and their application emphasized, and most of the related theories and formal proofs have been explained. The book places a strong emphasis on the physical meaning and applications of the concepts. Topics that have been selected are of high interest in the field. An attempt has been made to expose students and researchers to the most important topics and applications.

Organization of the Book

The book is organized so it can be used for teaching or for self-study. Chapter 1, “Tire Dynamics,” contains kinematics and coordinate frame transformation between different frames in tire, wheel, and vehicle body. It also includes the main theory behind combined tire force equations and their interactions. It also covers the vehicle load transfer by forward and lateral acceleration. Chapter 2, “Vehicle Planar Dynamics,” develops the equations of motion of a planar rigid vehicle, both bicycle and four-wheel models. Several examples for normal and critical maneuvers are presented. Chapter 3, “Vehicle Roll Dynamics,” follows the same method as the Chap. 2 to present the equations of motion of a roll rigid vehicle, also for both bicycle and four-wheel models. There are several new phenomena that appear only in the roll model. Several examples for normal and critical maneuvers are presented. Chapter 4, “Road Dynamics,” deals with the main concept of road design to help vehicles move safer and smoother.

Method of Presentation

This book uses a “*fact–reason–application*” structure. The “fact” is the main subject we introduce in each section. Then the reason is given as a “proof.” The application of the fact is examined in some “examples.” The “examples” are a very important part of the book as they show how to implement the “facts.” They also cover some other facts that are needed to expand the “fact.”

Prerequisites

Since the book is written for researchers and advanced graduate level students of engineering, the assumption is that users are familiar with matrix algebra, numerical analysis, differential equations, as well as principles of kinematics and dynamics. Therefore, the prerequisites are the fundamentals of kinematics, dynamics, vector analysis, matrix theory, numerical methods, and differential equations.

Unit System

The system of units adopted in this book is, unless otherwise stated, the international system of units (SI). The units of degree (deg) or radian (rad) are utilized for variables representing angular quantities.

Symbols

- Lowercase bold letters indicate a vector. Vectors may be expressed in an n -dimensional Euclidian space. Example:

$$\begin{matrix} \mathbf{r}, & \mathbf{s}, & \mathbf{d}, & \mathbf{a}, & \mathbf{b}, & \mathbf{c} \\ \mathbf{p}, & \mathbf{q}, & \mathbf{v}, & \mathbf{w}, & \mathbf{y}, & \mathbf{z} \\ \boldsymbol{\omega}, & \boldsymbol{\alpha}, & \boldsymbol{\epsilon}, & \boldsymbol{\theta}, & \boldsymbol{\delta}, & \boldsymbol{\phi} \end{matrix}$$

- Uppercase bold letters indicate a dynamic vector or a dynamic matrix, such as force and moment. Example:

$$\mathbf{F} \quad \mathbf{M}$$

- Lowercase letters with a hat indicate a unit vector. Unit vectors are not bold. Example:

$$\begin{matrix} \hat{i}, & \hat{j}, & \hat{k}, & \hat{u}, & \hat{u}, & \hat{n} \\ \hat{I}, & \hat{J}, & \hat{K}, & \hat{u}_\theta, & \hat{u}_\phi, & \hat{u}_\psi \end{matrix}$$

- Lowercase letters with a tilde indicate a 3×3 skew symmetric matrix associated with a vector. Example:

$$\tilde{\mathbf{a}} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

- An arrow above two uppercase letters indicates the start and end points of a position vector. Example:

$$\overrightarrow{ON} = \text{a position vector from point } O \text{ to point } N$$

- The length of a vector is indicated by a non-bold lowercase letter. Example:

$$r = |\mathbf{r}| \quad a = |\mathbf{a}| \quad b = |\mathbf{b}| \quad s = |\mathbf{s}|$$

- Capital letter B is utilized to denote a body coordinate frame. Example:

$$B(oxyz) \quad B(Oxyz) \quad B_1(o_1x_1y_1z_1)$$

- Capital letter G is utilized to denote a global, inertial, or fixed coordinate frame. Example:

$$G \quad G(XYZ) \quad G(OXYZ)$$

- Right subscript on a transformation matrix indicates the *departure* frames. Example:

$$R_B = \text{transformation matrix from frame } B(oxyz)$$

- Left superscript on a transformation matrix indicates the *destination* frame. Example:

$${}^G R_B = \text{transformation matrix from frame } B(oxyz) \\ \text{to frame } G(OXYZ)$$

- Capital letter R indicates rotation or a transformation matrix, if it shows the beginning and destination coordinate frames. Example:

$${}^G R_B = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Whenever there is no subscript or superscript, the matrices are shown in a bracket. Example:

$$[T] = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Left superscript on a vector denotes the frame in which the vector is expressed. That superscript indicates the frame that the vector belongs to, so the vector is expressed using the unit vectors of that frame. Example:

$${}^G \mathbf{r} = \text{position vector expressed in frame } G(OXYZ)$$

- Right subscript on a vector denotes the tip point that the vector is referred to. Example:

$${}^G \mathbf{r}_P = \text{position vector of point } P \\ \text{expressed in coordinate frame } G(OXYZ)$$

- Right subscript on an angular velocity vector indicates the frame that the angular vector is referred to. Example:

$$\boldsymbol{\omega}_B = \text{angular velocity of the body coordinate frame } B(oxyz)$$

- Left subscript on an angular velocity vector indicates the frame that the angular vector is measured with respect to. Example:

$${}_G \boldsymbol{\omega}_B = \text{angular velocity of the body coordinate frame } B(oxyz) \\ \text{with respect to the global coordinate frame } G(OXYZ)$$

- Left superscript on an angular velocity vector denotes the frame in which the angular velocity is expressed. Example:

$${}^{B_2} {}_G \boldsymbol{\omega}_{B_1} = \text{angular velocity of the body coordinate frame } B_1 \\ \text{with respect to the global coordinate frame } G, \\ \text{and expressed in body coordinate frame } B_2$$

Whenever the subscript and superscript of an angular velocity are the same, we usually drop the left superscript. Example:

$${}_G \boldsymbol{\omega}_B \equiv {}^G {}_G \boldsymbol{\omega}_B$$

Also for position, velocity, and acceleration vectors, we drop the left subscripts if it is the same as the left superscript. Example:

$${}^B {}_B \mathbf{v}_P \equiv {}^B \mathbf{v}_P$$

- Left superscript on derivative operators indicates the frame in which the derivative of a variable is taken. Example:

$$\frac{{}^G d}{dt} x \quad \frac{{}^G d}{dt} {}^B \mathbf{r}_P \quad \frac{{}^B d}{dt} {}^G \mathbf{r}_P$$

If the variable is a vector function, and also the frame in which the vector is defined is the same frame in which a time derivative is taken, we may use the following short notation,

$$\frac{{}^G d}{dt} {}^G \mathbf{r}_P = {}^G \dot{\mathbf{r}}_P \quad \frac{{}^B d}{dt} {}^B \mathbf{r}_P = {}^B \dot{\mathbf{r}}_P$$

and write equations simpler. Example:

$${}^G \mathbf{v} = \frac{{}^G d}{dt} {}^G \mathbf{r}(t) = {}^G \dot{\mathbf{r}}$$

- If followed by angles, lowercase c and s denote \cos and \sin functions in mathematical equations. Example:

$$c\alpha = \cos \alpha \quad s\varphi = \sin \varphi$$

- Capital bold letter \mathbf{I} indicates a unit matrix, which, depending on the dimension of the matrix equation, could be a 3×3 or a 4×4 unit matrix. \mathbf{I}_3 or \mathbf{I}_4 are also being used to clarify the dimension of \mathbf{I} . Example:

$$\mathbf{I} = \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- An asterisk \star indicates a more advanced subject or example that is not designed for undergraduate teaching and can be dropped in the first reading.

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