

SpringerBriefs in Economics

More information about this series at <http://www.springer.com/series/8876>

Dan S. Felsenthal · Hannu Nurmi

Voting Procedures Under a Restricted Domain

An Examination of the (In)Vulnerability
of 20 Voting Procedures to Five Main
Paradoxes

 Springer

Dan S. Felsenthal

Hannu Nurmi
Department of Philosophy, Contemporary
History and Political Science
University of Turku
Turku, Finland

ISSN 2191-5504

ISSN 2191-5512 (electronic)

SpringerBriefs in Economics

ISBN 978-3-030-12626-1

ISBN 978-3-030-12627-8 (eBook)

<https://doi.org/10.1007/978-3-030-12627-8>

Library of Congress Control Number: 2019933905

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2019

This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Voting paradoxes are a fascinating subject. Like all paradoxes, they are likely to invoke suspicion and disbelief; things that are supposed to work in an orderly, predictable and ‘nice’ fashion yield surprising, counterintuitive, and ‘nasty’ outcomes. Voting paradoxes have the distinction among the other paradoxes that they are related to man-made institutions. Hence, one would expect that they can be avoided by suitable re-drafting of institutions. It turns out, unfortunately, that basically all voting institutions are plagued with some paradoxes. Hence, the choice between voting procedures is a kind of balancing act where an effort to avoid certain types of paradoxes leads to the choice of procedures that are associated with other kinds of paradoxes.

In this booklet, we study a limited number of paradoxes and a pretty large, but still limited, number of procedures. This work complements two preceding booklets by the same authors, viz. *Monotonicity Failures Afflicting Procedures for Electing a Single Candidate* (2017) and *Voting Procedures for Electing a Single Candidate. Proving Their (In)Vulnerability to Various Voting Paradoxes* (2018). In the former text, we analyzed a class of paradoxes where, *ceteris paribus*, voters increase, or decrease, the support of some candidates thereby causing these candidates to be worse off, or better off, respectively. In the latter text, our focus was on a larger set of voting paradoxes. The setting in the two booklets was general: the choices ensuing from various procedures were considered in all possible preference environments. In other words, no restrictions whatsoever were imposed regarding how different are the voters’ opinions about the candidates or policy alternatives under examination. A procedure was classified as vulnerable to a voting paradox if even one distribution of opinions—however unlikely from a practical point of view—is found so that the paradox occurs. The present booklet takes a more nuanced look at the procedures and the paradoxes associated with them. More specifically, we focus on opinion distributions where the procedures under investigation initially lead to seemingly stable (in a specific sense) outcomes. We then study how this starting point is reflected in the vulnerability of the procedures to various voting paradoxes.

The present booklet contains eight chapters. Seven of these are joint works by the two authors, while Chap. 4 is authored by Dan S. Felsenthal alone. In the course of working on the chapters, we have incurred intellectual debts to many scholars of whom we would here like to mention Felix Brandt and Stefan Napel. This debt is gratefully acknowledged. We are most grateful to Dr. Martina Bihn—the Editorial Director of Business/Economics and Statistics at Springer—for her efficient handling of the production process and, above all, for her unfailing support and encouragement not only in the production of this booklet but of several earlier works of the authors. Thanks are also due to Springer staff member Judith Kripp for smooth cooperation over the months preceding the publication of this booklet, as well as to project manager Kokila Durairaj of Scientific Publishing Services, Chennai, India, and her team for making the publication of this booklet in its present form possible.

Jerusalem, Israel
Turku, Finland

Dan S. Felsenthal
Hannu Nurmi

The news of the passing away of Dan S. Felsenthal came just a few days after this booklet was sent to production. I lost a brilliant co-author and a good friend.

Hannu Nurmi

Contents

1	Introduction	1
	References	4
2	20 Voting Procedures Designed to Elect a Single Candidate	5
2.1	Non-ranked Voting Procedures	5
2.1.1	Plurality Voting (aka First Past the Post) Procedure	5
2.1.2	Plurality with Runoff Voting Procedure	6
2.1.3	Approval Voting (Brams & Fishburn, 1978, 1983)	6
2.1.4	Successive Elimination (Farquharson, 1969)	6
2.2	Ranked Voting Procedures That Are Not Condorcet-Consistent	7
2.2.1	Borda’s Count (Black, 1958; Borda, 1784)	7
2.2.2	Alternative Vote (aka Instant Runoff)	7
2.2.3	Coombs’s Method (Coombs, 1964, pp. 397–399; Coombs, Cohen, & Chamberlin, 1984; Straffin, 1980)	8
2.2.4	Negative Plurality Elimination Rule (NPER) (Lepelley, Moyouwou, & Smaoui, 2018)	8
2.2.5	Bucklin’s Method (Hoag & Hallett, 1926, pp. 485–491; Tideman, 2006, p. 203)	8
2.2.6	Range Voting (Smith, 2000)	9
2.2.7	Majority Judgment (Balinski & Laraki, 2007a, 2007b, 2011)	9
2.3	Ranked Voting Procedures That Are Condorcet-Consistent	10
2.3.1	The Minimax Procedure	10
2.3.2	Dodgson’s Procedure (Black, 1958, pp. 222–234; McLean & Urken, 1995, pp. 288–297)	10
2.3.3	Nanson’s Method (McLean & Urken, 1995, Chap. 14; Nanson, 1883)	11
2.3.4	Borda’s Elimination Rule (BER) (Baldwin, 1926)	11
2.3.5	Copeland’s Method (Copeland, 1951)	11
2.3.6	Black’s Method (Black, 1958, p. 66)	12

2.3.7	Kemeny's Method (Kemeny, 1959; Kemeny & Snell, 1960; Young, 1988, 1995; Young & Levenglick, 1978) . . .	12
2.3.8	Schwartz's Method (Schwartz, 1972, 1986)	13
2.3.9	Young's Method (Young, 1977)	13
	Exercises for Chapter 2	13
	Answers to Exercises of Chapter 2	14
	References	15
3	The (In)Vulnerability of 20 Voting Procedures to Lack of Monotonicity in a Restricted Domain	17
3.1	Introduction	17
3.2	The (In)Vulnerability of the Investigated Procedures to Upward Monotonicity Failure	20
3.2.1	Fixed Electorates	20
3.2.2	Variable Electorates	23
3.3	The (In)Vulnerability of the Investigated Procedures to Downward Monotonicity Failure	26
3.3.1	Fixed Electorates	26
3.3.2	Variable Electorates	29
3.4	Conclusions	36
	Exercises for Chapter 3	37
	Answers to Exercises of Chapter 3	38
	References	40
4	The (In)Vulnerability of 20 Voting Procedures to the Inconsistency Paradox (aka Reinforcement Paradox) in a Restricted Domain	41
4.1	Introduction	41
4.2	The (In)Vulnerability of the Various Procedures to the Inconsistency Paradox Under the Restricted Domain Assumption	42
4.2.1	The Condorcet-Consistent Procedures	42
4.2.2	Totally Invulnerable Non-Condorcet-Consistent Procedures	42
4.2.3	Plurality with Runoff and the Alternative Vote (AV) Procedures	43
4.2.4	Coombs's Procedure	44
4.2.5	The Negative Plurality Elimination Rule (NPER)	45
4.2.6	Bucklin's Procedure	45
4.2.7	The Majority Judgment (MJ) Procedure	46
4.3	Conclusion	47
	Exercises for Chapter 4	48
	Answers to Exercises of Chapter 4	48
	References	49

5 The (In)Vulnerability of 20 Voting Procedures to the No-Show Paradox in a Restricted Domain 51

5.1 Introduction 51

5.2 Assumptions and Definitions 53

5.3 Examples Demonstrating the Possibility of No-Show Paradox Under Eight Condorcet-Consistent and Three Non-Condorcet-Consistent Procedures When a Condorcet Winner Exists in the Initial Profile 53

5.3.1 Black’s Procedure 54

5.3.2 Kemeny’s Procedure 54

5.3.3 Nanson’s and the BER (Baldwin’s) Procedures 54

5.3.4 Successive Elimination Procedure 55

5.3.5 Young’s Procedure 55

5.3.6 Copeland’s Procedure 56

5.3.7 Dodgson’s Procedure 56

5.3.8 Coombs’s and the Negative Plurality Elimination Rule (NPER) Procedures 57

5.3.9 The Majority Judgment Procedure 57

5.4 Proofs Regarding the Impossibility of the No-Show Paradox Under Two Condorcet-Consistent and Three Non-Condorcet-Consistent Procedures When a Condorcet Winner Exists and Is Elected in the Initial Profile. 58

5.4.1 Minimax Procedure 58

5.4.2 Schwartz’s Procedure 59

5.4.3 The Plurality with Runoff Procedure 59

5.4.4 The Alternative Vote Procedure 60

5.4.5 Bucklin’s Procedure 60

5.5 Proofs Regarding the General Impossibility of the No-Show Paradox Under Four Non-Condorcet-Consistent Procedures 61

5.5.1 Plurality Voting 61

5.5.2 Approval Voting Procedure 61

5.5.3 Borda’s Procedure 62

5.5.4 Range Voting Procedure 62

5.6 Concluding Remarks 62

Exercises for Chapter 5 63

Answers to Exercises of Chapter 5 64

References 65

6 Which of 20 Voting Procedures Satisfy or Violate the Subset Choice Condition (SCC) in a Restricted Domain? 67

6.1 Introduction 67

6.2 The Subset Choice Condition 68

6.3 The Condorcet-Consistent Voting Procedures 69

6.4	Seven Non-Condorcet-Consistent Procedures Violating SCC Generally and Under the Restricted Domain Assumption	70
6.4.1	Plurality Voting	70
6.4.2	Plurality with Runoff	70
6.4.3	Alternative Vote	71
6.4.4	Coombs's Procedure	71
6.4.5	The Borda Count and the Negative Plurality Elimination Rule (NPER)	71
6.4.6	Bucklin's Procedure	72
6.5	Three Non-Condorcet-Consistent Procedures Which Satisfy SCC	72
6.6	Concluding Remarks	73
	Exercises for Chapter 6	73
	Answers to Exercises for Chapter 6	73
	References	74
7	The (In)Vulnerability of 20 Voting Procedures to the Preference Inversion Paradox in a Restricted Domain	75
7.1	Introduction	75
7.2	Condorcet-Consistent Procedures	78
7.2.1	Minimax	78
7.2.2	Young's Procedure	78
7.2.3	Dodgson's Procedure	78
7.2.4	Successive Elimination	79
7.2.5	Black's, Copeland's, Kemeny's, Nanson's, Baldwin's and Schwartz's Procedures	79
7.3	Ranked Non-Condorcet-Consistent Procedures	79
7.3.1	Plurality Voting	79
7.3.2	Approval Voting	80
7.3.3	Plurality with Runoff	80
7.3.4	Alternative Vote	81
7.3.5	Coombs's and the Negative Plurality Elimination Rule (NPER) Procedures	81
7.3.6	The Borda Count	81
7.3.7	Range Voting	81
7.3.8	Majority Judgment	82
7.3.9	Bucklin's Procedure	83
7.4	Concluding Remarks	84
	Exercises for Chapter 7	85
	Answers to Exercises of Chapter 7	86
	References	87
8	Summary	89
	Reference	92