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Algebraic and Differential Methods for Nonlinear Control Theory

Elements of Commutative Algebra and Algebraic Geometry

Springer

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ISSN 1559-7458 ISSN 1559-7466 (electronic)
Mathematical and Analytical Techniques with Applications to Engineering
ISBN 978-3-030-12027-6 ISBN 978-3-030-12025-2 (eBook)
https://doi.org/10.1007/978-3-030-12025-2

Library of Congress Control Number: 2018968389

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This book is written so the reader with little knowledge on mathematics and without experience in control theory can start from the beginning and go directly through the logical evolution of the topics. If the reader has advanced knowledge on mathematics, is fluent on abstract algebra, linear algebra, etc., will be able to read this book a little faster, even so, the reader must study the mathematics sections of the book, to make sure he is familiar with any special notation that we have presented. If the reader only knows about mathematics but has no experience in natural sciences or engineering, he must not be discouraged for the occasional use of examples and vocabulary of other fields (physical models or others). If the reader knows a little about control theory, maybe from previous courses of engineering or physics, it is possible that he will have to work slightly more at the beginning to get used to the ideas and notations in modern mathematics. When the reader is in a section that contains some mathematics that are new to him, it is required of the reader to study the examples, to do many exercises and to learn the new subject as if he has taken a course of it. At the end of the book, the reader will know not only control theory but also a little about useful mathematics. Whatever the previous experience the reader has, he will find in the book some topics that are needed to be learned for this particular area. As well, the book shows in a simple way some elementary concepts of mathematics such as basic algebraic structures (sets, groups, rings, fields, differential rings, differential fields, differential field extensions, etc., that is to say elements of commutative algebra and algebraic geometry in general) for the introduction to the theory of nonlinear control seen from a differential and algebraic point of view. Enough material has been included, rigorous proofs and illustrative exercises of each subject, as well as some examples and demonstrations are left to the reader for the good understanding of the chapters of the book. In the selection of material for this book, the authors intended to expound basic ideas and methods applicable to the study of differential equations in theory of control. The exposition is developed in such a way that the reader can omit passage that turn out to be difficult for him. The content of this book constitutes a tool that can be read by not only mathematicians, engineers and physicists but also all users of the theory of differential equations and theory of control. Control theory has become into a scientific discipline and of engineering that seems destined to have an impact in all aspects of modern society. First studied by mathematicians and engineers now it is increasingly present on areas of physics, economy, biology, sociology, psychology, etc. What is control theory and why is so important for a wide variety of specialities? Control theory is a set of concepts and techniques that are used to analyze and design various kinds of systems, independently from their physical nature and special functions. The most important aspect of control theory is the development of a quantitative model that describes the relation and interaction of a cause and effect between the variables of a system. This means that the language of control theory is mathematical and any serious attempt to learn control theory must be accompanied of mathematical precision and its comprehension. We have attempted, at writing this book, to make a pedagogic introduction to control theory that is accessible to readers with little background beyond linear and abstract elemental algebra and matrix theory while begin motivating for those mathematician readers with more sophisticated theoretical knowledge. As result, the book is more autonomous and less specialized. This book begins with the study of elementary set and map theory being a little bit abstract. Chapters 2 and 3 on groups theory and rings respectively are included because of their important relation to the linear algebra, the group of invertible linear maps (or matrices) and the ring of linear maps of a vector space being perhaps the most amazing examples of groups and rings. We deal with homomorphisms and Ideals as well as in Chap. 3 we mention some properties of integers. Also it is well known that group theory at its inception was deeply connected to algebraic equations. The success of the theory of groups in algebraic equations led to the hope that similar group-theoretic methods could be a powerful arsenal to attack the problems of differential equations. Still more, Ritt and Kolchin tackled the problem form the differential and algebraic point of view (Chap. 10 of control theory from point view of differential algebra). Chapter 4 is devoted to the matrices theory and the linear equations systems. Chapter 5 gives some definitions of permutations, determinants and inverse of a matrix. In Chap. 6 is tackled in notion of vector in real Euclidian space and in general vector space over a field, bases and dimension of a vector space as well as sums and direct sums. In Chap. 7 we treat with linear maps or linear transformations, Kernel and image of a linear map, dimensions of kernel and image, in addition, the application in linear control theory of some abstract theorems such as the concept of kernel, image and dimension of space are illustrated. Chapter 8 we shall consider the diagonalization of a matrix and the canonical form of Jordan. In Chap. 9 we attack elementary methods of solving differential equations and finally in Chap. 10 we treat the nonlinear control theory from the point of view of differential algebra.

Mexico City, Mexico

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Juan Javier Montesinos-García

Acknowledgements

To the memory of my father
who was the source of my teaching
Carlos Martínez Rosales.

To my wife and Sons
Marilen, Rafael and Juan Carlos.

To my mother and brothers
Virginia, Victor, Arturo, Carlos, Javier
and Marisela.

Je tiens à exprimer ma profonde gratitude
au Dr. Michel Fliess qui m'a initié au domaine
du contrôle non linéaire à l'aide, d'outils
tels que l'algèbre différentielle. Merci
également à mon ami Dr. Sette Diop
avec qui j'ai eu le plaisir de collaborer à certaines
publications. à eux ma plus sincère gratitude.

Rafael Martínez-Guerra

To my parents Teresa and Leopoldo.
In memory of my grandfather, Facundo.

Oscar Martínez-Fuentes

To my parents Irma and Javier,
whose support and love allowed me
to reach many goals.

To my aunt Rosario and uncle Clemente,
for all their teachings and support.

Juan Javier Montesinos-García

Contents

Notations and Abbreviations

$\mathbb{N}$	The Set of natural numbers
$\mathbb{Z}$	The Set of integers numbers
$\mathbb{Q}$	The Set of rational numbers
$\mathbb{R}$	The Set of real numbers
$\mathbb{C}$	The Set of complex numbers
$A, B, \ldots$	Capital letters represent arbitrary sets
$x, y, \ldots$	Lowercase letters represent elements of a set
$A \subset B$	A is subset of B
$x \in A$	x is element of A
$A \cup B$	The union of two sets
$A \cap B$	The intersection of two sets
A^c	Complement set of A
$A \backslash B$	Difference of sets A and B
$\emptyset$	Empty set
$\Longleftrightarrow$	Necessary and sufficient condition
$\forall$	For all
$\sim$	Equivalence relation
$a \equiv b \bmod n$	a is congruent with b module n
A/R	Quotient set
$*$	Binary operation for groups
$\det(A)$	The determinant of a square matrix $A \in \mathbb{R}^{n \times n}$
$\|A\|$	The determinant of a square matrix $A \in \mathbb{R}^{n \times n}$
$\operatorname{tr}(A)$	The trace of a matrix A
$A^\top$	The transpose of a matrix A
$\{\cdots\}$	Set
$(a_{ij})_{i,j}$	$m \times n$ matrix with entries a_{ij}, $1 \leq i \leq m, 1 \leq j \leq n$
$\operatorname{rank}(A)$	Rank of a matrix A
A^{-1}	Inverse of A
$\dot{y} = \frac{dy}{dt}$	First derivative of y with respect to t
E_λ	Eigenspace of E corresponding to eigenvalue λ

$\dim(V)$	Dimension of V
$\mathscr{L}(V, W)$	Space of linear transformations of V in W
Ker $\boldsymbol{\varphi}$	Kernel of $\boldsymbol{\varphi}$
diff trd°	Differential transcendence degree
$\square$	Designation of the end of a proof
$<(>)$	Less (greater) than
$\leq(\geq)$	Less (greater) than or equal to
$\forall$	For all
$P(n)$	A statement
$\cong$	Isomorphism
G/N	Quotient group
$K\langle u\rangle$	Differential field generated by the field K, u and its time derivatives
K/k	Differential field extension $k \subset K$