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Dieter Lüst · Ward Vleeshouwers

# Black Hole Information and Thermodynamics

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# Abstract

This SpringerBrief is based on a master's course on the black hole thermodynamics and the black hole information problem taught by Dieter Lüst during the summer term 2017 at the Ludwig-Maximilians-Universität in Munich; it was written by Ward Vleeshouwers. It provides a short introduction to general relativity, which describes gravity in terms of the curvature of space–time, and examines the properties of black holes. These are central objects in general relativity which arise when sufficient energy is compressed into a finite volume, so that even light cannot escape its gravitational pull. We will see that black holes exhibit a profound connection with thermodynamic systems. Indeed, by quantizing a field theory on curved backgrounds, one can show that black holes emit thermal (Hawking) radiation, so that the connection with thermodynamics is more than a formal similarity. Hawking radiation gives rise to an apparent conflict between general relativity and quantum mechanics known as the black hole information problem. If a black hole formed from a pure quantum state evaporates to form thermal radiation, which is in a mixed state, then the unitarity postulate of quantum mechanics is violated. We will examine the black hole information problem, which has plagued the physics community for over four decades, and consider prominent examples of proposed solutions, in particular, the string theoretical construction of the Tangherlini black hole, and the infinite number of asymptotic symmetries given by BMS transformations.

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