Mechanical Engineering Series

Series Editor

Francis A. Kulacki
Department of Mechanical Engineering
University of Minnesota
Minneapolis, Minnesota, USA

The Mechanical Engineering Series presents advanced level treatment of topics on the cutting edge of mechanical engineering. Designed for use by students, researchers and practicing engineers, the series presents modern developments in mechanical engineering and its innovative applications in applied mechanics, bioengineering, dynamic systems and control, energy, energy conversion and energy systems, fluid mechanics and fluid machinery, heat and mass transfer, manufacturing science and technology, mechanical design, mechanics of materials, micro- and nano-science technology, thermal physics, tribology, and vibration and acoustics. The series features graduate-level texts, professional books, and research monographs in key engineering science concentrations.

More information about this series at http://www.springer.com/series/1161

Amit Agrawal • Hari Mohan Kushwaha
Ravi Sudam Jadhav

Microscale Flow and Heat Transfer

Mathematical Modelling and Flow Physics

Springer

Amit Agrawal
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Mumbai, Maharashtra, India

Hari Mohan Kushwaha
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Mumbai, Maharashtra, India

Ravi Sudam Jadhav
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Mumbai, Maharashtra, India

ISSN 0941-5122 ISSN 2192-063X (electronic)
Mechanical Engineering Series
ISBN 978-3-030-10661-4 ISBN 978-3-030-10662-1 (eBook)
https://doi.org/10.1007/978-3-030-10662-1

Library of Congress Control Number: 2019932162

© Springer Nature Switzerland AG 2020
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG.
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

This book is dedicated to
past and current students of "Rarefied Gas Dynamics" and "Microfluidics" laboratories, IIT Bombay,
and
to our families.

Preface

This book has been written with the objective of making the readers familiar with the exciting developments on gaseous slip flow and heat transfer in microchannel. A large amount of work is being currently undertaken worldwide in these areas, with numerous potential applications. The subject is therefore topical and also particularly significant as it leads to the question about the validity of the Navier-Stokes equations, which is usually considered sacrosanct. It obviously leads to a subsequent question: if the Navier-Stokes equations are themselves not valid, then how do we model the flow and heat transfer? Although the answer to this last question is still not clear, it is important for the fluids and thermal community to first appreciate the limitations of the continuum approach (which leads to the Navier-Stokes equations), to better appreciate the ongoing search for the "beyond Navier-Stokes equations," and to test some of the available equations for their accuracy, before the larger objective of finding beyond the Navier-Stokes equations can be practically met.

The book is therefore organized in two parts: the first part is on the gaseous slip flow and heat transfer in microchannel (Chaps. 2 and 3). In the second part, we examine beyond the Navier-Stokes equations (Chaps. 5–7). Chapter 1 summarizes the characteristics of microscale flows and provides an introduction to the various modelling approaches available. Chapter 4 is a transition chapter between the two parts, where it is shown that simple extensions of the Navier-Stokes equations are not adequate. Recognizing that only analytical tool will not be adequate for studying flows in the slip and transition regimes, a brief overview to relevant numerical and experimental techniques is provided in Chap. 8. Finally, Chap. 9 summarizes our current understanding and provides suggestions for future research in this subject.

The Knudsen number is the most important parameter, and several known solutions with Knudsen number as an additional parameter are compiled in the first part of the book. Interesting observations on Knudsen minima and flow separation are presented. However, it should become apparent that our understanding of heat transfer at the microscale is not that sound, as several additional effects such as axial conduction, pressure work, conduction in the substrate, viscous dissipation, etc. coexist, but it is virtually not possible to treat all these effects together and obtain an analytical solution for even the simplest problems. Only few experimental and direct simulation Monte Carlo (DSMC) data exist, and they do not agree well with that obtained from alternative approaches.

A good portion of the second part of the book is devoted to deriving and understanding the Burnett and Grad equations. These two sets of equations form the most important "beyond the Navier-Stokes" equations and are generally referred to as "higher-order continuum transport equations." The study of these equations is important for further development of the subject, as they involve several novel concepts and represent important breakthroughs in the subject. The derivation of these equations has not often been repeated, and it is expected that the stepwise derivation presented here will invoke wide readership. Special effort has been made to make the text readable through the insertion of a large number of figures. The hope is that with the help of this book, it should now be possible to derive the Burnett and Grad equations in a graduate class. A few problems are solved to illustrate the type of solution obtained from these equations. In the derivation of some solutions presented here, some minor error in the original source was noticed which has been corrected here.

The first part of the book (Chaps. 1–3 along with Chaps. 8 and 9) should appeal to readers interested in understanding fundamental aspects of microscale flows and heat transfer, while the second part (Chaps. 4–7) is primarily for slightly advanced readers interested in understanding equations beyond the Navier-Stokes equations.

Mumbai, India Amit Agrawal
Mumbai, India Hari Mohan Kushwaha
Mumbai, India Ravi Sudam Jadhav
September 2018

Acknowledgments

We are grateful to all the students who have worked in the “Rarefied Gas Dynamics” and “Microfluidics” laboratories at IIT Bombay, including those who were involved in purely theoretical or numerical aspects of the problem. It is they who slowly and patiently uncovered the various fine points of the subject, which finally culminated in the form of this book. We are also grateful to Prof. K. Muralidhar (IIT Kanpur) and Prof. F. Kulacki (University of Minnesota), the editors of this series, for first placing their trust in us for writing this book and then patiently waiting for it while we went past several deadlines. The book would not have resulted without the coaxing of Prof. Muralidhar to write on this subject. While preparing the book, we gained from the comments of Prof. Atul Sharma (IIT Bombay).

Funding for this line of investigation has been provided by the Indian Institute of Technology Bombay (IITB), Indian Space Research Organisation (ISRO), Department of Science and Technology (DST), and recently by the Department of Atomic Energy (DAE) under its prestigious scheme—DAE-SRC Outstanding Investigator Award, awarded to the first author.

Contents

Nomenclature

Alphabetical

$\mathbf{c}$	molecular velocity, $\frac{m}{s}$
c_s	speed of sound, $\frac{m}{s}$
c_x, c_y, c_z	components of molecular velocity $\mathbf{c}$, $\frac{m}{s}$
c_p	specific heat at constant pressure, $\frac{J}{kg\ K}$
c_v	specific heat at constant volume, $\frac{J}{kg\ K}$
$\mathbf{C}$	peculiar velocity, $\frac{m}{s}$
C_1, C_2	Slip coefficients
C_x, C_y, C_z	components of peculiar velocity $\mathbf{C}$, $\frac{m}{s}$
d	molecular diameter, m
D_h	hydraulic diameter, m
e	specific energy, $\frac{J}{kg}$
$\mathbf{F}$	external force, $\frac{kg\ m}{s^2}$
f	particle distribution function, $\frac{s^3}{m^6}$
f_M	Maxwell-Boltzmann distribution, $\frac{s^3}{m^6}$
h	convective heat transfer coefficient, $\frac{W}{m^2\ K}$
k	thermal conductivity, $\frac{W}{m\ K}$
k_B	Boltzmann constant, $\frac{J}{K}$

L	characteristic length scale, m
m	mass of a molecule, kg
$\dot{m}$	mass flow rate, $\frac{kg}{s}$
n	number density, $\frac{1}{m^3}$
p	hydrostatic pressure, $\frac{N}{m^2}$
P_{ij}	pressure tensor, $\frac{N}{m^2}$
p_{ij}	divergence-free part of pressure tensor P_{ij}, $\frac{N}{m^2}$
$\mathbf{q}$	heat flux vector, $\frac{W}{m^2}$
R	specific gas constant ($= k_B/m$), $\frac{J}{kg.K}$
r, θ, z	cylindrical coordinate system
r, θ, ϕ	spherical coordinate system
t	time, s
T	absolute temperature, K
T_m	bulk mean temperature, K
T_w	wall temperature, K
$\mathbf{u}$	bulk velocity vector, $\frac{m}{s}$
u, v, w	components of bulk velocity $\mathbf{u}$, $\frac{m}{s}$
u_s	velocity of gas at the wall, $\frac{m}{s}$
u_w	wall velocity, $\frac{m}{s}$
$\bar{v}$	most probable speed, $\frac{m}{s}$
$\mathbf{x}$	position vector, m
x, y, z	Cartesian coordinates in physical space

Greek symbols

α	thermal diffusivity, $\frac{m^2}{s}$
γ	specific heat ratio, $\frac{c_p}{c_v}$
λ	mean free path, m
δ_{ij}	Kronecker delta

ϵ	specific internal energy, $\frac{J}{kg}$
μ	dynamic viscosity, $\frac{N\ s}{m^2}$
μ_2	second coefficient of viscosity, $\frac{N\ s}{m^2}$
μ_v	coefficient of bulk viscosity, $\frac{N\ s}{m^2}$
ν	kinematic viscosity, $\frac{m^2}{s}$
Φ	viscous dissipation, $\frac{kg}{m\ s^3}$
ρ	mass density, $\frac{kg}{m^3}$
σ_T	thermal accommodation coefficient
σ_v	tangential momentum accommodation coefficient
σ_{ij}	viscous stress tensor, $\frac{N}{m^2}$
τ_{ij}	stress tensor $(= -P_{ij})$, $\frac{N}{m^2}$

Non-dimensional numbers

Br	Brinkman number, $Br = \frac{\mu u^2}{k(T_w - T_m)}$
Ec	Eckert number, $Ec = \frac{u^2}{c_p(T_w - T_m)}$
Kn	Knudsen number, $Kn = \frac{\lambda}{L}$
Ma	Mach number, $Ma = \frac{u}{\sqrt{\gamma RT}}$
Nu	Nusselt number, $Nu = \frac{hD_h}{k}$
Pe	Peclet number, $Pe = \frac{\rho u c_p D_h}{k}$
Pr	Prandtl number, $Pr = \frac{\mu c_p}{k}$
Re	Reynolds number, $Re = \frac{\rho u D_h}{\mu}$

Abbreviations

CFD	computational fluid dynamics
DSMC	direct simulation Monte Carlo
MEMS	micro-electro-mechanical systems
TMAC	tangential momentum accommodation coefficient