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Applied and Numerical Harmonic Analysis

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Landscapes of Time–Frequency Analysis

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Their continuous support is our strength

ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time–frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time–frequency and time-scale analysis*
Numerical partial differential equations	*Wavelet theory*

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time–frequency–scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

College Park

John J. Benedetto
Series Editor
University of Maryland

Preface

The first international conference *Aspects of Time–Frequency Analysis* "ATFA17" took place during 5–7 June 2017 at the Politecnico di Torino. It was a major international scientific event gathering many of the brightest stars in harmonic analysis and its applications. This meeting was jointly organized by Dipartimento di Matematica (Università di Torino), Dipartimento di Scienze Matematiche (Politecnico di Torino) and the Numerical Harmonic Analysis Group (NuHAG, Vienna).

The Organizing Committee consisted of Paolo Boggiatto, Elena Cordero, Alessandro Oliaro (University di Torino), Maurice de Gosson, Hans Feichtinger (Universität Wien), Enrico Magli, Fabio Nicola, and Anita Tabacco (Politecnico di Torino).

The financial support was granted by local funds from the Dipartimento di Matematica (Università di Torino) and Dipartimento di Scienze Matematiche (Politecnico di Torino); partial support was provided by Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni—GNAMPA (INDAM).

Topics included function spaces, time–frequency analysis and Gabor analysis, sampling theory and compressed sensing, mathematical signal processing, microlocal analysis, pseudodifferential and Fourier integral operators, numerical harmonic analysis, abstract harmonic analysis, and applications of harmonic analysis to quantum mechanics. This wide range of topics illustrates well the broadness of the scope of ATFA17. The given talks formed the heart of the conference and provided ample opportunity for fruitful discussions and led in some cases to new collaborations.

It is our duty and pleasure to thank all participants for their contributions to the conference program.

The present volume gathers written texts from invited participants, our choice covering the full range of the conference topics. It thus reflects well the spirit of ATFA17.

Organizing the volume and reminding late contributors was a challenging process, may Elena Cordero be praised for her patience and tenacity and careful proofreading!

We would also like to thank the Proceedings team for having invested so much time in very dedicated and professional work. The ATFA17 proceeding is a credit to a large group of people, and everybody should be proud of the outcome. The success of this conference means that we can now envisage with confidence the next event ATFA19 to be held in Turin in June 2019. We are sure that it will be as interesting and enjoyable as its predecessor.

Vienna, Austria Maurice de Gosson
August 2018

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2. Dipartimento di Matematica, Università di Torino, via Carlo Alberto 10, 10123 Torino, Italy,
3. GNAMPA—Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni, INDAM, Italy.

One of the successes of ATFA17 (Aspects of Time–Frequency Analysis) has been the dynamic participation of graduate students and postdocs, pure and applied mathematicians, and scientists of other disciplines. We have been fortunate to be able to provide travel and living expenses to many participants due to the funds from the aforementioned institutions.

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