Advances in Mechanics and Mathematics

Volume 42

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Mathematical Models of Higher Orders

Shells in Temperature Fields

Springer

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ISSN 1571-8689 ISSN 1876-9896 (electronic)
Advances in Mechanics and Mathematics
ISBN 978-3-030-04713-9 ISBN 978-3-030-04714-6 (eBook)
https://doi.org/10.1007/978-3-030-04714-6

Library of Congress Control Number: 2018962138

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

The content of this monograph exemplifies the vast range of mathematical models of nonlinear dynamics and statics (to some extent) of continuous mechanical structural members based mainly on consideration of beams, plates, and shells, with a particular emphasis on shells interacting with internal and external thermal fields. We consider models that are well known, those that we have modified, and those that are new.

Though there exist numerous books devoted to the study of the dynamics of structural members, the majority of investigations deal with linear modeling of coupled problems of thermoelasticity. One of the unique features of our book relies on consideration of a heat transfer equation in a 3D formulation, whereas the shell equations yielded by the Hamilton variational principle exhibit a different type and dimension (hyperbolic and hyperbolic–parabolic). This book offers a valuable methodological approach to the state of the art of the above-mentioned classical plate/shell mathematical models, including the kinematic model of the first-order (Kirchhoff–Love) and second- order (Timoshenko) approximations, as well as the mixed Grigolyuk–Chulkov model. Analysis of multilayer-shell nonlinear dynamics and related stability problems “in the large” is based on the mathematical models introduced by Timoshenko (second-order approximation), Sheremetev–Pelekh–Levinson–Reddy (third-order approximation), Grigolyuk–Kulikov (hyperbolic model), and the novel models derived with the help of the stationary variant of the “projectional conditions” of a shell motion and the model with ε-regularization.

The main thrust of this monograph points out the need for further investigations of the classical problem of shell dynamics consisting of mathematical modeling, derivation of nonlinear PDEs, and finding their solutions based on new and effective numerical techniques, strongly supported by useful theorems and their proofs.

Another challenging feature of the book embraces its engineering aspect devoted to carrying out the optimal design of deformable mechanical constructions, highlighting the problems of diffraction (transmission) or decomposition of plates and shells. In general, the problem of diffraction can be reduced to boundary value problems in spaces consisting of a few different materials, and it is governed by boundary value problems associated with differential equations with discontinuous coefficients. We show how to solve such nonclassical problems in the framework of appropriately chosen phases with adequate norms and/or configuration spaces.

This monograph introduces a new class of generalized problems of diffraction in the theory of shells based on the fundamental variational equations of the thermomechanics of shallow shells. In addition, the mathematical theories for nonclassical geometrically nonlinear models and the design of inhomogeneous shallow shells imply a coherent development of reliable computational programs in finding solutions whose existence has been formally proved.

In particular, the proved existence of a generalized solution has allowed us to give a theoretical basis for the validation of the Faedo–Galerkin method used in higher approximations (the proof embraces coupled/uncoupled problems of thermoelasticity and Timoshenko hypotheses and their modifications).

This monograph is enriched by numerical algorithms for solving the governing 3D and 2D (hyperbolic and hyperbolic–parabolic) PDEs. Both the Faedo–Galerkin and finite difference methods (FDM) have been employed to second-order accuracy to solve the problem entirely, i.e., treating it as a well-posed problem of infinite dimension in contrast to what is widely met in literature, namely strong truncation, reducing the problem to only a few degrees of freedom and hence having limited application. Furthermore, the reader should find our original approach attractive, for it is aimed at solving static problems of thermoelasticity in which the treatment of the dynamics is based on the setup (relaxation) method (the latter approach is very efficient in comparison with the classical techniques of solving static problems).

Finally, this book offers new results in the nonlinear dynamics of the objects studied, including regular (periodic and quasiperiodic) and irregular (timing chaos and spatiotemporal chaos) vibrations and their bifurcations. The latter study is enriched by wavelet-based (in contrast to the classical Fourier transform) analysis aimed at detecting and tracing the time evolution of the frequency spectra and estimation of the Lyapunov exponents based on the neural network approach.

The book consists of seven chapters. Chapter 1 offers an introduction to the problems considered in the book, with emphasis on both the importance of a rigorous mathematical treatment and application-oriented mathematical modeling. It provides a description of the state of the art of problems devoted to the nonclassical mathematical modeling of structural members as well as highlighting the gaps in research and challenges that one faces in developing novel models, which is important for obtaining reliable and computationally acceptable solutions to the governing nonlinear partial differential equations.

Chapter 2 is devoted to derivation of the mathematical models of nonlinear dynamics of plates and shallow shells under internal/external temperature fields. The variational formulation gives PDEs of different dimensions and different types.

The problem is studied within the first-order kinematic Kirchhoff–Love model, taking into account von Kármán geometric nonlinearity, a physical nonlinearity, and heat transfer processes. The governing PDEs are solved by the Faedo–Galerkin method in higher approximations and the finite difference method to second-order accuracy. A wide class of problems of nonlinear vibrations of shells and shells has been solved in a comprehensive way with numerous analyses of case studies.

Nonclassical mathematical models and stability problems of multilayer orthotropic thermoelastic shells within modified Timoshenko-type hypotheses are studied in Chapter 3, which includes the development of the construction of improved mathematical models aimed at a rigorous study of nonlinear plates and shells under the action of temperature fields. This chapter also illustrates the reliability of boundary value problems of the models analyzed and discusses the problem of static stability of multilayer orthotropic shells based on numerical investigations.

Chapter 4 deals with the general problem of diffraction in the theory of plates/shells interacting locally with temperature fields. PDEs of different types and dimensions are derived from the Hamilton/Onsager variational equations. The existence of a generalized solution is proved, and the use of the Faedo–Galerkin method is validated. Timoshenko, Kirchhoff–Love, and Grigolyuk–Chulkov models are considered, and coupled problems of thermoelasticity exhibited by mixed-form PDEs in displacements are illustrated and solved.

Chapter 5 deals with stability of flexible shallow shells under transversal load and heat flow in which a coupling of thermal and deformation fields is neglected. Dynamic pre- and postcritical behaviors of spherical and cylindrical shells are studied. It is shown that an increase in the nondimensional heat transfer coefficient implies an increase in thermal stresses and deflection amplitudes of the studied shells, among other effects.

Chapter 6 presents methods and algorithms for economical (in the sense of computational time) numerical investigations of the stability of multilayer flexible orthotropic shells "in the large" under temperature fields within the models of Timoshenko second-order approximation, the Sheremetev-Pelekh–Reddy–Levinson third-order approximations, the Grigolyuk–Kulikov models and their modifications, and the model with ε-regularization. In the latter case, a theorem regarding the existence of a general solution is proved. Algorithms devoted to the difference approximation of differential operators appearing in the governing PDEs of asymmetric packing of multilayer shells are developed. Numerous results dealing with the obtained "load-deflection" stability curves are presented and discussed, also with regard to application of the different mathematical models.

Chapter 7 focuses on the analysis of chaotic vibrations of closed cylindrical shells under local transversal load and temperature field within the first-order Kirchhoff–Love approximation model. A novel scenario of transition from regular to chaotic shell dynamics is detected and illustrated. The influence of the damping coefficient and the common action of the temperature field and the local harmonic load is investigated, among other things.

This book is written for graduate and doctoral students in mechanical and civil engineering, applied mathematicians and physicists, as well as for engineers engaged in the study of nonlinear dynamics of structural members. It may be helpful also for academics, researchers, and professionals interested in a rigorous and comprehensive study of modeling nonlinear phenomena governed by PDEs.

We greatly appreciate the help and support of the Springer Editors, Elizabeth Leow and Dahlia Fisch while writing the book.

Finally, we would like to acknowledge a support of the Russian Science Foundation RSF No 16-11-10138 grant.

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Contents