

Control Engineering

Series Editor

William S. Levine
Department of Electrical and Computer Engineering
University of Maryland
College Park, MD
USA

Editorial Advisory Board

Richard Braatz
Massachusetts Institute of Technology
Cambridge, MA
USA

Graham Goodwin
University of Newcastle
Australia

Davor Hrovat
Ford Motor Company
Dearborn, MI
USA

Zongli Lin
University of Virginia
Charlottesville, VA
USA

Mark Spong
University of Texas at Dallas
Dallas, TX
USA

Maarten Steinbuch
Technische Universiteit Eindhoven
Eindhoven, The Netherlands

Mathukumalli Vidyasagar
University of Texas at Dallas
Dallas, TX
USA

Yutaka Yamamoto
Kyoto University
Kyoto, Japan

Ashish Tewari

Optimal Space Flight Navigation

An Analytical Approach

Ashish Tewari
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur
IIT-Kanpur, India

ISSN 2373-7719 ISSN 2373-7727 (electronic)
Control Engineering
ISBN 978-3-030-03788-8 ISBN 978-3-030-03789-5 (eBook)
<https://doi.org/10.1007/978-3-030-03789-5>

Library of Congress Control Number: 2018961705

Mathematics Subject Classification: 70F05, 70F07, 70F10, 49N05, 49N25, 49J15, 49K15, 49J30, 37N05, 34B10

© Springer Nature Switzerland AG 2019

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This book is published under the imprint Birkhäuser, www.birkhauser-science.com by the registered company Springer Nature Switzerland AG

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This book is primarily written to consolidate optimal space flight navigation theory as a discipline. There are several excellent textbooks on optimal control theory and equally good books on orbital space mechanics, but the application of optimal control theory to orbital mechanics is mainly confined to research articles. Over the last several decades, the research literature on optimal space flight navigation has become voluminous and has fragmented into many esoteric subdisciplines. However, a monograph which organizes this content and vitalizes it for contemporary application is hitherto lacking.

Optimal space flight navigation is a problem of practical interest and has been so ever since humans imagined journeying into space. While visionaries such as Tsiolkovsky and Goddard helped in taking the first steps by developing rocketry, it required enthusiasts to devise fuel-optimal trajectories for practical space missions. The vision of Walter Hohmann, a civil engineer, revealed in his 1925 book *Die Erreichbarkeit der Himmelskörper (The Accessibility of Celestial Bodies)* is one of the most useful and commonly applied space flight maneuvers—the Hohmann transfer. As recently as in 2014, the Hohmann transfer was successfully applied to efficiently send India’s first mission to Mars. While arising as an elegant idea in Hohmann’s mind, the optimality of the Hohmann transfer can now be rigorously proved in various ways, including by optimal control theory. Similarly, the straightforward and insightful application by D.F. Lawden of the erstwhile nascent optimal control theory to the space navigation problem produced in 1963 in the monograph entitled *Optimal Trajectories for Space Navigation* has guided many researchers since then. The first footsteps on the Moon would not have appeared if NASA’s engineers (including Richard Battin who passed away in 2014) had not devised simple but efficient techniques to guide the lunar module on a flat thrusting trajectory to a safe landing. Battin in his treatise *An Introduction to the Mathematics and Methods of Astrodynamics* reminisces about how he came up with the velocity-to-be-gained and cross-product steering as simple but practical guidance strategies for space flight in the era of primitive computer technology. He writes of the enormous buildings required to house the computer mainframes of the day with only very modest computing power. Perhaps the total computational

resources present on board *Apollo 11* could be surpassed by those of today's pocket calculator. The feat performed by those pioneering guidance engineers can only be appreciated by the complexity of their task—to send people to the Moon and to bring them successfully back to the Earth. The design and analysis of practical lunar and interplanetary trajectories could be conducted in the analytical framework established by Victor Szebehely in his 1967 masterpiece, *Theory of Orbits: The Restricted Problem of Three Bodies*.

Rather than highlighting the computational (and often unrealizable) solutions of mathematically complex problems, this book emphasizes the analytical approach to optimal space navigation. The need for simplicity and practicability of guidance methods is to be contrasted with the unfortunate emphasis which has been placed these days on increasing the complexity of control schemes. Multilevel iterative algorithms must be solved in real-time computations and therefore have unresolved convergence issues which prevent applications to actual missions. It should thus be asked: Is more sophistication always better, and does it lead to any advantage in the practical sense? I think a simpler guidance solution must be sought, wherever possible, due to its reliable implementation in a realistic situation, which has been amply demonstrated by the analytical guidance methods. That is why even the latest space flight missions are based upon analytical techniques.

The complex interplanetary and asteroid (or comet) flyby and rendezvous missions are analytically derived from the application of optimal control theory to multi-body dynamics, such as the optimal transfers between halo and quasi-halo orbits and Lissajous trajectories using the stable and unstable manifolds of the restricted three-body problem. Studying such optimal paths often imparts new insights into the problem of multi-body transfers, which would be lacking in a purely numerical search for suboptimal solutions. In a similar vein, the problem of orbiting and landing on an irregularly shaped body with an uncertain gravity field (such as an asteroid or a comet) can be solved out by time-optimal and fuel-optimal methods, as has been demonstrated in many practical missions.

The thought of writing this book arose from the course AE-777 (Optimal Space Flight Control), which I have taught for the past several years and for which a single textbook containing all the relevant material was unavailable. Therefore, the book is designed to be used as a textbook in a course on optimal space flight at the graduate and senior undergraduate levels. The first three chapters give a basic introduction to the topic, namely, optimal control theory (Chaps. 1 and 2) and orbital mechanics with impulsive transfers (Chap. 3). Chapter 4 covers optimal transfers in a spherical gravity field with continuous, unconstrained acceleration inputs, whereas Chap. 5 extends the treatment to trajectory optimization with bounded acceleration inputs. Finally, Chap. 6 introduces the reader to the advanced topics of optimal transfer in time-varying gravity fields, including multi-body transfers. Unfortunately, due to the vast scope of these topics, it is not possible to do them full justice in an introductory text. Hence, references to research articles are provided for the advanced topics. The end-of-chapter exercises test the understanding of the basic concepts, whereas several references are provided for undertaking research and for advanced applications. A basic knowledge of control systems and dynamics is

assumed of the reader, which can be supplemented by textbooks on these topics, such as those previously published by me (e.g., *Modern Control Design, and Atmospheric and Space Flight Dynamics, Automatic Control of Atmospheric and Space Flight Vehicles*).

It is a pleasure to offer this manuscript for publication with Birkhäuser. I would like to thank the editorial and production staff at Birkhäuser for their invaluable assistance, especially Bill Levine, Benjamin Levitt, Samuel DiBella, and Christopher Tominich. I would finally like to thank my wife Prachi and daughter Manya for their patience during the preparation of this manuscript.

Kanpur, India
September 2018

Ashish Tewari

Contents

1	Introduction	1
1.1	Optimal Control	1
1.2	Space Vehicle Guidance	3
2	Analytical Optimal Control	7
2.1	Introduction	7
2.2	Optimization of Static Systems	8
2.2.1	Static Equality Constraints	11
2.2.2	Inequality Constraints	18
2.3	Dynamic Equality Constraints and Unbounded Inputs	20
2.4	Special Boundary Conditions	26
2.4.1	Fixed Terminal Time	26
2.4.2	Free Terminal Time	28
2.5	Sufficient Condition for Optimality	31
2.6	Pontryagin's Minimum Principle	31
2.7	Hamilton–Jacobi–Bellman Formulation	32
2.8	Time-Invariant Systems	34
2.9	Linear Systems with Quadratic Cost Functions	35
2.10	Illustrative Examples	43
2.11	Singular Optimal Control	55
2.11.1	Generalized Legendre–Clebsch Necessary Condition	58
2.11.2	Jacobson's Necessary Condition	67
2.12	Numerical Solution Procedures	69
2.12.1	Shooting Method	70
2.12.2	Collocation Method	71
	Exercises	71
3	Orbital Mechanics and Impulsive Transfer	77
3.1	Introduction	77
3.2	Keplerian Motion	78
3.2.1	Reference Frames of Keplerian Motion	80

3.2.2	Time Equation.....	83
3.2.3	Lagrange's Coefficients.....	86
3.3	Impulsive Orbital Transfer.....	88
3.3.1	Minimum Energy Transfer.....	92
3.4	Lambert's Transfer.....	95
3.4.1	Stumpff Function Method.....	96
3.4.2	Hypergeometric Function Method.....	99
3.5	Optimal Impulsive Transfer.....	101
3.5.1	Coasting Arc.....	102
3.5.2	Hohmann Transfer.....	105
3.5.3	Outer Bi-elliptical Transfer.....	110
	Exercises.....	112
4	Two-Body Maneuvers with Unbounded Continuous Inputs.....	115
4.1	Introduction.....	115
4.2	A Motivating Example.....	115
4.3	Equations of Motion.....	117
4.4	Optimal Low-Thrust Orbital Transfer.....	120
4.4.1	Coplanar Orbital Transfer.....	122
4.4.2	Plane Change Maneuver.....	122
4.4.3	General Orbital Transfer.....	123
4.5	Variational Model.....	124
4.6	Optimal Regulation of Circular Orbits.....	127
4.6.1	Coplanar Regulation with Radial Thrust.....	128
4.6.2	Coplanar Regulation with Tangential Thrust.....	131
4.7	General Orbital Tracking.....	132
4.8	Basic Guidance with Continuous Inputs.....	138
4.9	Line-of-Sight Guidance.....	139
4.10	Cross-Product Steering.....	145
4.11	Energy-Optimal Guidance.....	148
4.12	Hill-Clohessy-Wiltshire Model.....	151
	Exercises.....	155
5	Optimal Maneuvers with Bounded Inputs.....	159
5.1	Introduction.....	159
5.2	Optimal Thrust Direction.....	159
5.2.1	Constant Acceleration Bound.....	166
5.2.2	Bounded Exhaust Rate.....	166
5.3	Time-Invariant Gravity Field.....	168
5.4	Null-Thrust Arc in Central Gravity Field.....	169
5.4.1	Inverse-Square Gravity Field.....	172
5.5	Intermediate-Thrust Arc.....	173
5.6	Lawden's Spiral.....	174
5.7	Powered Arcs.....	180

- 5.8 Linearization Relative to a Circular Orbit 189
 - 5.8.1 Out-of-Plane Rendezvous 193
 - 5.8.2 Coplanar Rendezvous 196
- Exercises 197
- 6 Flight in Non-spherical Gravity Fields 199**
 - 6.1 Introduction 199
 - 6.2 Gravity Field of a Non-spherical Body 201
 - 6.2.1 Gravity of an Axisymmetric Planet 201
 - 6.2.2 Gravity Field of an Oblate Planet 207
 - 6.2.3 Gravity of an Irregular Body 208
 - 6.3 Circular Restricted Three-Body Problem 208
 - 6.3.1 Lagrangian Points and Stability 213
 - 6.3.2 Hamiltonian Formulation and Jacobi’s Integral 216
 - 6.4 Special Three-Body Trajectories 219
 - 6.4.1 Perturbed Orbits About a Primary 219
 - 6.4.2 Free-Return Trajectories Between the Primaries 220
 - 6.5 Optimal Low-Thrust Three-Body Transfer 222
 - 6.6 Missions Around Collinear Lagrangian Points 232
 - 6.6.1 Motion About the Collinear Points 234
 - 6.6.2 Lindstedt–Poincaré Method for Halo Orbits 237
 - 6.7 Numerical Computation of Halo Orbits and Manifolds 242
 - 6.7.1 Monodromy Matrix 243
 - 6.7.2 Stable and Unstable Manifolds 246
 - 6.8 Optimal Station Keeping around Collinear Points 249
 - 6.8.1 Impulsive Control 249
 - 6.8.2 Continuous Thrust Control 250
- Answers to Selected Exercises 253**
- References 263**
- Index 267**