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Władysław Narkiewicz

The Story of Algebraic Numbers in the First Half of the 20th Century

From Hilbert to Tate

Springer

Władysław Narkiewicz
University of Wrocław
Wrocław, Poland

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To the memory of my wife

Preface

The aim of this book is to give a survey of results in the theory of algebraic numbers achieved in the first half of the twentieth century and may be viewed as a companion to my previous book *Rational Number Theory in the 20th Century* in which the part of number theory dealing with rational numbers has been treated. It is an attempt to fulfil the wish of H. S. Vandiver expressed in 1960 in his paper [4185], and perhaps it might be helpful in preventing rediscoveries.

Chapter 1 gives a concise presentation of the beginnings of the theory of algebraic numbers. One finds here first a description of the work on special cases of algebraic integers done by Gauss, Dirichlet and Eisenstein, followed by Kummer's work on cyclotomic fields. Then the creation of the general theory by Kronecker and Dirichlet is treated, and the chapter concludes with a short description of the related work of other mathematicians, including Hermite, Minkowski, Frobenius and Stickelberger.

In Chap. 2 one finds a presentation of the work of Hilbert, who in his report on algebraic numbers summarized the state of their theory at the end of the nineteenth century, as well of Hensel, who created p-adic and $\mathfrak{p}$-adic numbers, which turned out to be an indispensable tool in future research. In the last part of the chapter the first steps towards creation of the class-field theory, characterizing Abelian extensions of algebraic number fields, are described.

Chapter 3 covers the first twenty years of the twentieth century. In the first section we present its central subject, the use of analytic methods in the theory of algebraic numbers. This has been initiated by Landau, who established the Prime Ideal Theorem giving asymptotics for the number of prime ideal with bounded norms. The next big achievement was Hecke's proof of the continuation of the Dedekind zeta-function to a meromorphic function on the plane, and the study of several generalizations of Dirichlet L-functions to number fields, developed by him and Landau. The second section presents the results dealing with the algebraic structure, and the last section is devoted to other results achieved in the beginning of the twentieth century.

The central themes of Chap. 4 are the creation of the modern ideal theory by Emmy Noether, and the establishment of fundamental theorems of class-field theory by Takagi and Artin. Also various other questions were considered at that time, for example the first results in the additive theory of algebraic numbers obtained by Rademacher.

In Chap. 5 we present first the progress in the study of the structure of number fields, the central subject being the existence of normal and normal integral bases, and then consider some additive questions, mainly on sums of squares. The next section concentrates on the simplification of the class-field theory by Hasse and Chevalley, and the following sections concern i.a. the class-number and class-group of quadratic fields, the question of the existence of the Euclidean algorithm, the distribution of algebraic integers on the complex plane and infinite extensions of number fields. Chapter 6 covers The Forties, the main results being obtained by Brauer and Siegel.

In all chapters one will find also some selected information about the subsequent developments of the arising problems.

I am very grateful to my friends Kálman Győry and Andrzej Schinzel for reading the draft of the book and providing several comments and suggestions. I thank also the referees of the book for several important hints.

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Wrocław, Poland Władysław Narkiewicz

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