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Jiashi Yang

# An Introduction to the Theory of Piezoelectricity

Second Edition

 Springer

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# Preface

The previous edition of this book, published in 2005, was based on the lecture notes for a graduate course offered at the former Department of Engineering Mechanics of the University of Nebraska–Lincoln since 1998. The course was intended to provide graduate students performing research in electromechanical materials and devices with the basic theoretical aspects of the continuum modeling of electroelastic interactions in solids. As such, the scope of the book was, and still is, somewhat limited. However, this second edition includes multiple additions and removals of sections and has gone through a reorganization of chapters and sections. In particular, the derivation of the electric body force, couple, and power, which is one of the fundamental building blocks for the construction of the nonlinear theory of electroelasticity, is presented using both integral and differential approaches based on Tiersten’s two-continuum model. They are also derived using the polarized particle model and the effective polarization charge model in Appendices 1 and 2, respectively.

Although most of the book is devoted to the linear theory of piezoelectricity, the book begins with a chapter on the nonlinear theory of electroelasticity. It is hoped that this will be helpful for a deeper understanding of the theory of piezoelectricity, which is the linearization of the nonlinear theory about a natural state with zero fields. The presentation of the linear theory of piezoelectricity begins with Chap. 2, and readers who are not interested in nonlinear electroelasticity can begin with Chap. 2 directly. Chapters 3, 4, and 5 are all based on the linear theory of piezoelectricity, which assumes infinitesimal deviations from an ideal reference state in which there are no pre-existing mechanical and/or electric fields (initial or biasing fields). The presence of biasing fields makes a material behave like a different one and renders the linear theory of piezoelectricity invalid. The behavior of electroelastic bodies under biasing fields can be described by the linear theory for infinitesimal incremental fields superposed on finite biasing fields in Chap. 6, which is the linearization of the nonlinear theory of electroelasticity about a finite bias. Chapter 7 discusses a few effects that are external to the general theoretical framework developed in Chap. 1. Chapter 8 presents a few examples of the application of the theory of piezoelectricity.

For the linear theory of piezoelectricity, the notation of *IEEE Standard on Piezoelectricity* is followed; however, notation varies among researchers regarding the nonlinear theory of electroelasticity. In this book, the notation of H. F. Tiersten is followed. The literature on the theory of piezoelectricity is abundant, so only those that are directly used in the book are listed as references.

This book mainly covers the exact three-dimensional theory of piezoelectricity. For approximate one- and two-dimensional theories of thin piezoelectric structures of beams, plates, and shells as well their applications in piezoelectric devices, please see the following books by this author published by World Scientific: *The Mechanics of Piezoelectric Structures*, 2006; *Analysis of Piezoelectric Devices*, 2006; *Antiplane Motions of Piezoceramics and Acoustic Wave Devices*, 2010; and *Vibration of Piezoelectric Crystal Plates*, 2013.

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