

UNITEXT - La Matematica per il 3+2

Volume 113

Editor-in-chief

Alfio Quarteroni

Series editors

L. Ambrosio

P. Biscari

C. Ciliberto

C. De Lellis

M. Ledoux

V. Panaretos

W. J. Runggaldier

More information about this series at <http://www.springer.com/series/5418>

Lorenzo Peccati · Mauro D'Amico
Margherita Cigola

Maths for Social Sciences

 Springer

Lorenzo Peccati
Department of Decision Sciences
Bocconi University
Milan, Italy

Margherita Cigola
Department of Decision Sciences
Bocconi University
Milan, Italy

Mauro D'Amico
Department of Decision Sciences
Bocconi University
Milan, Italy

Additional material to this book can be downloaded from <http://extras.springer.com>.

ISSN 2038-5714 ISSN 2532-3318 (electronic)
UNITEXT - La Matematica per il 3+2
ISSN 2038-5722 ISSN 2038-5757 (electronic)
ISBN 978-3-030-02335-5 ISBN 978-3-030-02336-2 (eBook)
<https://doi.org/10.1007/978-3-030-02336-2>

Library of Congress Control Number: 2018958490

© Springer Nature Switzerland AG 2018

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This textbook is the natural evolution of some teaching notes prepared for the course on “Quantitative Methods - Module 1, Mathematics” in the “Bachelor in International Government” program at Bocconi University. While many books are available on statistics for the social sciences, few try to cover mathematics for the social sciences. This textbook represents such an attempt, the aim being to present some mathematical tools of value in the social sciences through a number of examples.

These mathematical tools are:

- Linear algebra;
- Differential calculus;
- Integral calculus;
- Dynamic systems.

The style of presentation is informal, and most of the examples are calibrated with reference to the application areas.

We are grateful to the referees, who examined the first version of this textbook and made insightful suggestions, and to our colleagues Ross MacMillan and Massimo Marinacci for useful suggestions regarding the contents.

The sole responsibility for any error, whether in the English or in the mathematics, resides with the authors.

The solutions of the exercises provided in the textbook are available at the following link <http://extras.springer.com>.

Milan, Italy
July 2018

Lorenzo Peccati
Mauro D’Amico
Margherita Cigola

Contents

| | | |
|----------|---|----|
| 1 | Linear Algebra | 1 |
| 1.1 | Vectors and Matrices | 1 |
| 1.1.1 | Introductory Examples | 1 |
| 1.2 | Operations on Vectors | 4 |
| 1.2.1 | Vector Representation in \mathbb{R}^2 and in \mathbb{R}^3 | 4 |
| 1.2.2 | Vector Addition | 5 |
| 1.2.3 | Scalar Multiplication | 7 |
| 1.2.4 | Linear Combination of Vectors | 8 |
| 1.2.5 | Linear Dependence/Independence of Vectors | 10 |
| 1.3 | Matrices | 16 |
| 1.3.1 | Types of Matrices | 16 |
| 1.3.2 | Operations on Matrices: An Inventory | 18 |
| 1.3.3 | Transposition | 19 |
| 1.3.4 | Matrix Addition | 20 |
| 1.3.5 | Multiplication of a Scalar by a Matrix | 21 |
| 1.3.6 | Multiplication of Matrices | 22 |
| 1.4 | Determinants | 41 |
| 1.4.1 | Notion and Computation | 41 |
| 1.4.2 | How to Invert a Matrix Using Determinants | 50 |
| 1.4.3 | Determinants and “Viability” of a Leontief System | 51 |
| 1.5 | Rank of a Matrix | 52 |
| 1.6 | Statistical Applications of Linear Algebra | 58 |
| 1.7 | Linear Applications | 67 |
| 1.8 | Linear Algebraic Systems | 71 |
| 1.8.1 | A Special Case: Cramer’s Systems | 72 |
| 1.8.2 | The General Case | 74 |
| 1.9 | Applications to Networks | 86 |
| 1.10 | Some Complements on Square Matrices | 87 |
| 1.11 | Exercises | 93 |

| | | |
|----------|--|-----|
| 2 | Differential Calculus | 99 |
| 2.1 | What's a Function | 99 |
| 2.1.1 | Intervals | 100 |
| 2.1.2 | Easy Functions | 101 |
| 2.1.3 | Elementary Functions | 101 |
| 2.1.4 | Continuous Functions | 110 |
| 2.1.5 | An Annoying Detail and a Tribute to L.D. Landau | 112 |
| 2.1.6 | The Small o Algebra | 114 |
| 2.1.7 | Some Rankings | 115 |
| 2.2 | Local Behavior and Global Behavior | 118 |
| 2.2.1 | Local Behavior, Derivative and Differential | 118 |
| 2.2.2 | Notation for Derivatives | 125 |
| 2.2.3 | Derivative and Differential: What's the Most Important Notion | 126 |
| 2.2.4 | The Computation of Derivatives, also of Order >1 | 128 |
| 2.3 | What's a Function of a Vector | 152 |
| 2.3.1 | Graphic Representation of a Function of $n \geq 2$ Variables | 155 |
| 2.3.2 | How Big is a Vector? | 157 |
| 2.3.3 | Derivatives of Functions of a Vector | 158 |
| 2.3.4 | Unconstrained Extrema for Functions $f: \mathbb{R}^n \rightarrow \mathbb{R}$ | 163 |
| 2.3.5 | Constrained Extrema | 170 |
| 2.3.6 | The General Case | 178 |
| 2.4 | Exercises | 192 |
| 3 | Integral Calculus | 197 |
| 3.1 | Integrals and Areas | 197 |
| 3.2 | Fundamental Theorem of Integral Calculus | 198 |
| 3.3 | Antiderivative Calculus | 201 |
| 3.3.1 | Integration by Parts | 202 |
| 3.3.2 | Integration by Substitution | 203 |
| 3.4 | An Immediate Application: Mean and Expected Values | 204 |
| 3.4.1 | Expectation and the Law of Large Numbers | 206 |
| 3.4.2 | Density Function and Distribution Function | 207 |
| 3.4.3 | Discrete Distributions | 209 |
| 3.5 | Frequency/Probability Density Functions: Some Cases | 210 |
| 3.5.1 | A Special Distribution | 211 |
| 3.6 | People Survival | 214 |
| 3.7 | Exercises | 217 |
| 4 | Dynamic Systems | 221 |
| 4.1 | Introduction | 221 |
| 4.2 | Local Information: The Motion Law | 224 |
| 4.2.1 | Discrete Time | 224 |

| | | |
|-----------------------------|---|------------|
| 4.2.2 | Continuous Time | 226 |
| 4.2.3 | Motion Law of a DS | 228 |
| 4.2.4 | Autonomous Systems | 229 |
| 4.3 | Extracting Info from a Motion Law | 232 |
| 4.4 | Classic Approach | 233 |
| 4.4.1 | Linear Discrete Systems | 234 |
| 4.4.2 | About Some Special Discrete Systems | 246 |
| 4.4.3 | Continuous-Time Systems | 249 |
| 4.4.4 | Continuous Systems: Separable Equations | 250 |
| 4.4.5 | Continuous Systems: Linear Differential Equations of the First Order | 253 |
| 4.4.6 | An Interesting Socio-demographic Model/1 | 254 |
| 4.4.7 | Linear Continuous Systems | 260 |
| 4.5 | Numerical Approach | 282 |
| 4.5.1 | Discrete Systems | 282 |
| 4.5.2 | Continuous Systems | 283 |
| 4.6 | Qualitative Approach | 285 |
| 4.6.1 | Equilibria: Notion and General Systems | 285 |
| 4.6.2 | How to Find Equilibria for Autonomous Systems? | 287 |
| 4.6.3 | Nature of an Equilibrium Point | 289 |
| 4.7 | A Newcomer: The Phase Diagram | 290 |
| 4.7.1 | Notion | 290 |
| 4.7.2 | Equilibria in a Phase Diagram | 293 |
| 4.7.3 | Behaviors Revealed by a Phase Diagram | 295 |
| 4.7.4 | Continuous Systems | 307 |
| 4.8 | Some Politically Relevant Applications | 309 |
| 4.8.1 | Consensus Diffusion | 310 |
| 4.8.2 | Arab Springs | 315 |
| 4.8.3 | Growth and Demographic Trap | 318 |
| 4.9 | Military Applications | 321 |
| 4.9.1 | War | 321 |
| 4.9.2 | Guerrilla | 322 |
| 4.10 | USA Against USSR: An Old Story | 324 |
| 4.10.1 | An Electoral Application | 329 |
| 4.11 | Epidemics | 332 |
| 4.11.1 | The B Model | 332 |
| 4.11.2 | The R Model | 333 |
| 4.11.3 | The RI Model | 334 |
| 4.11.4 | The RIV Model | 335 |
| 4.12 | Exercises | 337 |
| References | | 341 |
| Index | | 343 |

About the Authors

Lorenzo Peccati was born in 1944 and is an Emeritus Professor of Mathematics at Bocconi University. He was the Editor of the European Journal of Operational Research for 10 years. He has authored over 140 publications in national and international journals as well as several books.

Mauro D'Amico was born in 1963 and is an Instructor in Mathematics at Bocconi University. He has an extensive experience in teaching both university and business school courses. In addition, he has written several textbooks in Italian.

Margherita Cigola was born in 1961 and is an Associate Professor of Mathematics at Bocconi University. Her research focuses on optimization theory, and she has an extensive experience in teaching applied mathematics.