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A Sampling of Remarkable Groups

Thompson's, Self-similar, Lamplighter, and Baumslag-Solitar

Birkhäuser

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For Gilbert

Preface

Note to Students

If you read this text carefully, working out the examples as you go along before reading their solutions, you will be rewarded with some intriguing ideas while gaining group theoretic knowledge. The groups we have chosen to explore in this work are both interesting and important (we discuss why in the individual chapters). Feel free to jump to whichever chapter captures your attention; the chapters need not be read in order. However, that being said, it may be necessary for you to refer back to the Preliminaries chapter (► Chap. 1) or an outside source for some concepts or definitions (these are indicated by italics at their first mention in a chapter). Several concepts and definitions recur throughout many chapters and are developed carefully. Other interesting ideas are mentioned only briefly as possibilities for outside research and definitions of terms are not always given; feel free to ignore these tangential ideas if you wish. More assistance from the relevant literature can also be found in the Bibliography at the end of the text.

The material in this book is meant to be challenging in that it relies heavily on concepts from earlier mathematical courses you have taken, it uses notation you may not have encountered before (and may see expressed differently elsewhere), and it will introduce you to a wide range of proofs. Do not be deterred! Exercises that may prove particularly challenging are marked with an *.

For researchers and students with a more extensive mathematical background, this book is valuable for its introduction to interesting groups and for its references for further study and open questions in the sections titled "Topics for Further Exploration."

Note to Instructors

This work is intended to be valuable as a supplement to a first course in Abstract Algebra or as a text or supplement for a higher level course. Its chapters are crafted so that they may be assigned individually (with references to ► Chap. 1) or all together. Through the careful reading and solving of the exercises in this text, students will get a chance to synthesize material learned in earlier mathematical courses and be exposed to a wide range of proofs, from simple to complex. This book is also filled with examples and exercises, which lend themselves well to research/honors projects.

The authors claim full responsibility for any errors appearing in these notes.

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The authors at the Joint Mathematics Meeting 2015 in San Antonio, TX

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