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Kurusch Ebrahimi-Fard · María Barbero Liñán  
Editors

# Discrete Mechanics, Geometric Integration and Lie–Butcher Series

DMGILBS, Madrid, May 2015

 Springer

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# Preface

Geometric numerical integration is a rather important research topic in numerical analysis of differential equations. In the introductory chapter of this volume, two distinguished mathematicians, Arieh Iserles and Reinout Quispel, explore recent and ongoing developments, as well as new research directions in geometric integration methods for differential equations. The collection of manuscripts following Iserles' and Quispel's contribution display a combination of research and overview chapters including detailed presentations of many of the mathematical tools necessary in the areas of geometric integration theory, nonlinear systems theory, and discrete mechanics. The scope and high quality of this volume is maybe best exemplified by briefly mentioning the topics it contains. Many mechanical systems evolve on Lie groups, that is why Lie group integrators are essential for numerically solving differential equations. A comprehensive overview on Lie group integrators is provided by Brynjulf Owren. The algebraic, geometric, and computational aspects relevant to numerical integration methods, such as Lie–Butcher series and word series algorithms, are described extensively in the following chapters by Munthe-Kaas and Føllesdal, Murua and Sanz-Serna, Ebrahimi-Fard and Mencattini, and Casas. The contribution by Duffaut Espinosa, Ebrahimi-Fard, and Gray explores interconnections of nonlinear systems with a view towards discretisation. The following chapters by Bogfjellmo, Dahmen, and Schmeding, Barbero Liñán and Martín de Diego, Vermeeren, and Verdier are shorter and address more specific research questions, with the exception of the paper by Bogfjellmo et al., which also includes a timely overview of Lie theoretic and Hopf algebraic aspects relevant to geometric numerical integration. Indeed, a common thread underlying those works is the fruitful use of modern algebraic and combinatorial structures common to those topics.

The contributions are written in a self-contained style to make the volume accessible to a broader audience, including in particular researchers and graduate students interested in theoretical and applied aspects in geometric integration theory, nonlinear control theory, and discrete mechanics.

These chapters are based on extended lectures and research talks presented at the international “Brainstorming Workshop on New Developments in Discrete Mechanics, Geometric Integration and Lie–Butcher Series”. The event took place at the Instituto de Ciencias Matemáticas (ICMAT) in Madrid, Spain, and was one of the main activities organised by the Norwegian–Spanish NILS–ABEL 2014–2015 research project “Discrete Mechanics, Geometric Integration and Lie–Butcher Series”. The two partners of the NILS–ABEL project (Bergen–Madrid) were very eager to consult with experts (Elena Celledoni and Brynjulf Owren) from the Norwegian University of Science and Technology (NTNU) in Trondheim. In fact, we were convinced that combining the expertise from researchers from these three institutions and including other invited participants would certainly lead to a substantial boost of the perspectives of this mathematics research project. This was one of the main motivations to organise this brainstorming workshop back in 2015. The meeting brought together senior experts as well as young researchers, from Germany, Norway, Spain, and the USA. Its central aim was to provide a platform for discussing theoretical and applied aspects of computational solutions of differential equations describing dynamical systems in natural sciences and technology as well as nonlinear control systems. We particularly appreciate Profs. Iserles and Quispel for their valuable contribution to this volume. Although they could not join us in the workshop, they were very enthusiastic in preparing the introductory chapter.

Last but not least, this volume would not have been possible without the commitment of all the speakers in the workshop. They prepared excellent expositions which made this event rather successful. The event received funding from the EEA grant provided by Norway, Iceland, Liechtenstein, and Spain (NILS), as well as from the Fundación BBVA, the ICMAT Severo Ochoa Excellence Programme, and Universidad Carlos III de Madrid. We also thank the referees for helping us in preparing this volume and the ICMAT for providing the facilities and the human resources to make the event a success.

Trondheim, Norway  
Madrid, Spain

Kurusch Ebrahimi-Fard  
María Barbero Liñán

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