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Geometric Flows and the Geometry of Space-time

Birkhäuser

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ISSN 2522-0969 ISSN 2522-0977 (electronic)
Tutorials, Schools, and Workshops in the Mathematical Sciences
ISBN 978-3-030-01125-3 ISBN 978-3-030-01126-0 (eBook)
https://doi.org/10.1007/978-3-030-01126-0

Library of Congress Control Number: 2018960861

Mathematics Subject Classification (2010): 53C44, 53C50, 53C27, 53C29

This book is published under the imprint Birkhäuser, www.birkhauser-science.com by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

A major goal in mathematics as well as in physics has been and still is to understand the geometry of space and time. Developments in both subjects have fruitfully influenced each other over the history of science. The formulation of general relativity by Einstein would not have been possible without the concepts of (semi-) Riemannian geometry that had emerged with the visionary ideas of Riemann in the previous century. Conversely, ideas from general relativity influenced mathematical research and the study of Einstein's equation is one of today's major topics in geometric analysis.

Similarly, the development of more recent areas of theoretical physics, such as string theory, is deeply connected to the study of geometric problems in mathematics, such as the study of metrics of special holonomy. It turned out that geometric flows are also of great importance in the interplay between mathematics and physics; e.g., the Riemann Penrose inequality has been shown by Huisken and Ilmanen using the inverse mean curvature flow.

This volume is based on a summer school and workshop entitled "Geometric flows and the geometry of space-time" held at the University of Hamburg in September 2016. The aim of this event was to provide a forum where physicists and mathematicians can exchange ideas and where graduate students and young researchers get the opportunity to learn about recent developments at the intersection of mathematics and physics.

It brought together around 60 participants with mathematical and physical backgrounds. The speakers were Lars Andersson, Helga Baum, Spiros Cotsakis, Pau Figueras, Gary Gibbons, Mark Haskins, Jason Lotay, Thomas Leistner, Jan Metzger, and Oliver C. Schnürer.

Out of these 10 speakers, 7 gave two talks where the first one was more of an introductory nature and the other one was more focused on actual research. These talks covered a broad variety of topics, ranging from special holonomy metrics to various concepts of mass in general relativity and the numerical and analytic study of black hole space-times.

Moreover, three of the speakers gave minicourses where each of them had a total length of 180 min. One minicourse was more of a physical nature and was held

by Gary Gibbons about the theory of black holes. The other two lecture courses were more of a mathematical nature. One course was held by Oliver C. Schnürer about geometric flows and focused in particular on mean curvature flow. The other course held by Helga Baum was about special holonomy and parallel spinors in Lorentzian geometry. In addition, we had two related talks about Cauchy problems for Lorentzian manifolds of special holonomy by Thomas Leistner.

This volume consists of two articles. The first is based on the mathematical lecture course by Oliver C. Schnürer and the second on the mathematical lecture course by Helga Baum extended by results presented in the lectures by Thomas Leistner.

Another volume based on the third lecture course about the theory of black holes is planned. The papers are written for graduate students and researchers with a general background in geometry and in the theory of partial differential equations, who want to get acquainted with these central subjects of modern geometry. We hope this volume will be helpful and inspiring.

Hamburg, Germany
July 2018

Vicente Cortés
Klaus Kröncke
Jan Louis

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