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Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group

Springer

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Preface

The theory of mean periodic functions is a subject which goes back to works of Littlewood, Delsarte, John and that has undergone a vigorous development in recent years. There has been much progress in a number of problems concerning local aspects of spectral analysis and spectral synthesis on homogeneous spaces. The study of these problems turns out to be closely related to a variety of questions in harmonic analysis, complex analysis, partial differential equations, integral geometry, approximation theory, and other branches of contemporary mathematics. The present book describes recent advances in this direction of research.

Symmetric spaces and the Heisenberg group are an active field of investigation at the moment. The simplest examples of symmetric spaces, the classical 2-sphere $\mathbb{S}^2$ and the hyperbolic plane $\mathbb{H}^2$, play familiar roles in many areas in mathematics. The Heisenberg group H^n is a principal model for nilpotent groups, and results obtained for H^n may suggest results that hold more generally for this important class of Lie groups. The purpose of this book is to develop harmonic analysis of mean periodic functions on the above spaces.

The book consists of four parts. Part I is devoted to symmetric spaces and related questions. After some general considerations in Chap. 1, rank one symmetric spaces play here a privileged role. There is a number of books that are characteristic of this subject from the abstract point of view. Our text differs at this point and is based on realizations of rank one spaces as domains in Euclidean space. The aim of such an approach is twofold: on the one hand, in this way we hope to contribute towards a better visualization and a better handling of these spaces; on the other hand, in addition to their intrinsic interest, these realizations will play an important role in our study of transmutation operators on rank one compact symmetric spaces in Part II. The exposition in Chaps. 2–5 of Part I has on occasion been used as a textbook for first-year graduate students without background in Lie group theory.

Part II develops the transmutation operator theory. We define appropriate analogues of the Abel–Radon transform and give a treatment of their basic properties. The generalized homomorphism property is the crucial one; it relates the mean periodicity on the spaces in question to that on $\mathbb{R}^1$ and allows many proofs in Parts III and IV to be carried out by reduction to the one-dimensional case.

Parts III and IV deal with the theory of mean periodic functions on domains of Euclidean spaces, Riemannian symmetric spaces, and the Heisenberg group. Attention was focused on Fourier-type decompositions and on the “hard analysis" problems that could be attacked with them: structure of zero sets of mean periodic functions and modern versions of John’s support theorem, local analogues of the Schwartz fundamental principle, the problem of mean-periodic continuation, Hörmander-type approximation theorems on domains without the convexity assumption, explicit reconstruction formulae in the deconvolution problem, Zalcman-type two-radii problems on domains of symmetric spaces of arbitrary rank, local versions of the Brown–Schreiber–Taylor theorem on spectral analysis and their symmetric space analogues, and so on. The difficulty in studying the above varies with spaces. Nevertheless, in all cases almost all results are the best possible, i.e., give answers to all questions which naturally arise in the topic and present a complete picture of the corresponding phenomenon. The proofs given are "minimal" in the sense that they involve only such concepts and facts which are indispensable for the essence of the subject. We shall have nothing to say in this book on the mean-periodicity on domains of compact symmetric spaces of higher ranks (except the case of the whole space) but hope that the methods that we develop will prove useful in this connection.

Each part begins with a summary and ends with comments. The reader will find here not only historical notes and further results but also many challenging conjectures and open problems and the invitation to work in this exiting field. The authors hope that the exposition in the book will be comprehensible to anyone who knows the elements of functional analysis and possesses sufficient perseverance in overcoming purely logical difficulties. All the necessary information is given in the text with references to the sources.

Some of the material in this book has been the subject of lectures delivered by the authors over a number of years. We have received helpful comments and suggestions from many colleagues; of these we mention R. Trigub, V. Zastavnyĭ, D. Zaraisky, A. Grishin, V. Ryazanov, V. Belyĭ, L. Ronkin, and B. Kotlyar. We thank them all.

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Finally, our thanks go to Springer for the realization of this monograph.

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Contents