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Liuping Wang

Model Predictive
Control System Design
and Implementation
Using MATLAB[®]

 Springer

Liuping Wang, PhD
School of Electrical and Computer Engineering
RMIT University
Melbourne, VIC 3000
Australia

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Series Editors

Professor Michael J. Grimble, Professor of Industrial Systems and Director
Professor Michael A. Johnson, Professor (Emeritus) of Control Systems and Deputy Director

Industrial Control Centre
Department of Electronic and Electrical Engineering
University of Strathclyde
Graham Hills Building
50 George Street
Glasgow G1 1QE
United Kingdom

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K7L 3N6
Canada

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4 Engineering Drive 3
Singapore 117576

Professor (Emeritus) O.P. Malik
Department of Electrical and Computer Engineering
University of Calgary
2500, University Drive, NW
Calgary, Alberta
T2N 1N4
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Hong Kong

Professor G. Olsson
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221 00 Lund
Sweden

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Pennsylvania State University
0329 Reber Building
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3335 Engineering II
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The University of Kitakyushu
Faculty of Environmental Engineering
1-1, Hibikino, Wakamatsu-ku, Kitakyushu, Fukuoka, 808-0135
Japan

In memory of my parents

Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies..., new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

Today's control software and technology offers the potential to implement more advanced control algorithms but often the preferred strategy of many industrial engineers is to design a robust and transparent process control structure that uses simple controllers. This is one reason why the PID controller remains industry's most widely implemented controller despite the extensive developments of control theory; however, this approach of structured control can create limitations on good process performance. One such limitation is the possible lack of a coordinator within the hierarchy that systematically achieves performance objectives. Another is the omission of a facility to accommodate and handle process operational constraints easily. The method of model predictive control (MPC) can be used in different levels of the process control structure and is also able to handle a wide variety of process control constraints systematically. These are two of the reasons why MPC is often cited as one of the more popular advanced techniques for industrial process applications.

Surprisingly, MPC and the associated receding horizon control principle have a history of development and applications going back to the late 1960s; Jacques Richalet developed his predictive functional control technique for industrial application from that time onward. Work on using the receding horizon control concept with state-space models can be identified in the literature of the 1970s, and the 1980s saw the emergence first of dynamic matrix control and then, towards the end of the decade, of the influential generalised predictive control technique.

This field continues to develop and the *Advances in Industrial Control* monograph series has several volumes on the subject. These include *Applied*

Predictive Control by S. Huang, K.K. Tan and T.H. Lee (ISBN 978-1-85233-338-6, 2002), *Fuzzy Logic, Identification and Predictive Control* by J.J. Espinosa, J.P.L. Vandewalle and V. Wertz (ISBN 978-1-85233-828-2, 2005) and *Advanced Control of Industrial Processes* by P. Tatjewski (ISBN 978-1-84628-634-6, 2007). In our related series, *Advanced Textbooks in Control and Signal Processing*, we have published *Model Predictive Control* by E.F. Camacho and C. Bordons (2nd edition, ISBN 978-1-85233-694-3, 2004), and *Receding Horizon Control* (ISBN 978-1-84628-024-5, 2005) by W.H. Kwon and S. Han .

To the above group of books we are now able to add this monograph, *Model Predictive Control System Design and Implementation Using MATLAB[®]*, by Liuping Wang. Professor Wang aims to provide both the industrial and the academic reader with a direct but graded route into understanding MPC as used in the solution of industrial control problems. The interleaved exposition, and MATLAB[®] tutorials, allow the reader to work through a structured introduction to the design and implementation of MPC and use some related tools to condition, tune and test the control design solutions.

Some features of MPC that makes it worthy of study as an industrial control technique include:

- the technique uses simple concepts;
- the controller tuning can be packaged for ease of use;
- the technique can be used in either supervisory or primary control modes; and
- constraint handling is naturally accommodated by the method, and can be packaged for automated systematic constraint setup.

Professor Wang's book illustrates these issues and uses a small set of theoretical tools to great effect; these tools include the exponential weighting of signals, weights to achieve a prescribed degree of stability, re-parameterisation of the feedback using orthogonality principles, Laguerre and Kautz basis functions, and quadratic programming. The book is structured so that discrete methods are considered first, and these are followed by continuous-time system techniques. Over the course of the book, the MATLAB[®] interludes result in readers constructing their own libraries of MPC routines that can be used in other control problems. Towards the end of the book, Professor Wang demonstrates the use of the MPC algorithms in some application studies. These range from a motor control application to the control of a food extruder process and these studies illustrate both the software and hardware aspects of the solutions.

The book's "hands-on" approach is expected to appeal to a wide readership ranging from the industrial control engineer to the postgraduate student in the process and control disciplines. Both will undoubtedly find the MATLAB[®] demonstrations of the control concepts an invaluable tutorial route to understanding MPC in practice.

Industrial Control Centre
Glasgow
Scotland, UK
2008

M.J. Grimble
M.A. Johnson

Preface

About this Book

Model predictive control (MPC) has a long history in the field of control engineering. It is one of the few areas that has received on-going interest from researchers in both the industrial and academic communities. Four major aspects of model predictive control make the design methodology attractive to both practitioners and academics. The first aspect is the design formulation, which uses a completely multivariable system framework where the performance parameters of the multivariable control system are related to the engineering aspects of the system; hence, they can be understood and ‘tuned’ by engineers. The second aspect is the ability of the method to handle both ‘soft’ constraints and hard constraints in a multivariable control framework. This is particularly attractive to industry where tight profit margins and limits on the process operation are inevitably present. The third aspect is the ability to perform on-line process optimization. The fourth aspect is the simplicity of the design framework in handling all these complex issues.

This book gives an introduction to model predictive control, and recent developments in design and implementation. Beginning with an overview of the field, the book will systematically cover topics in receding horizon control, MPC design formulations, constrained control, Laguerre-function-based predictive control, predictive control using exponential data weighting, reformulation of classical predictive control, tuning of predictive control, as well as simulation and implementation using MATLAB[®] and SIMULINK[®] as a platform. Both continuous-time and discrete-time model predictive control is presented in a similar framework.

Development of this Book

There are several aspects of the developments that may be of interest to the reader.

Theory

This book was originally planned as a research monograph to cover the methodologies of predictive control using orthonormal basis functions. Design of predictive control using time-domain functions is not new. It was proposed by Richalet in the 1970s and has been successfully used in various control engineering applications (see Richalet *et al.*, 1978, Richalet, 1987, 1993, 2000). However, the design of predictive control that uses orthonormal functions is new, particularly the approaches that use the sets of exponential orthonormal functions such as Laguerre functions and Kautz functions. With my background in system identification, I naturally saw the link between system identification and predictive control from the perspective of modelling of the control trajectory. Once the predictive control problem is formulated as that of modelling the future control trajectory, both continuous-time and discrete-time predictive control is solved in the same framework. Both have direct links to the classical linear quadratic regulator (LQR) in continuous time and discrete time when using a sufficiently long prediction horizon. The key difference between predictive control and LQR is that the predictive control solves the optimization problem using a moving time horizon window whilst LQR solves the same problem within a fixed window. The advantages of using a moving time horizon window include the ability to perform real-time optimization with hard constraints on plant variables.

As we know, the majority of industrial control systems have integral action (*e.g.*, PID controllers). This integral functionality has also been embedded in the classical predictive control systems such as generalized predictive control (GPC) and dynamic matrix control (DMC) (see Clarke *et al.*, 1987, Cutler and Ramaker, 1979). In this book, in order to have an integral function in the MPC algorithms, the design models are embedded with integrators. The formulation of the design models was inspired by the state-space approach given in Ricker's paper (Ricker, 1991). By doing so, similar to GPC and DMC, the optimized control trajectory is either the increment of the control signal (discrete-time case) or the derivative of the control signal (continuous-time case). The added benefit is the simplified implementation procedure, where less steady-state plant information is required.

One of the well-known problems in classical predictive control is its numerical problem when the prediction horizon is large (for example, Rossiter *et al.*, 1998). This was inevitable because the model used for design contained an integrating mode. To overcome this problem, the design model needs to be asymptotically stable. Using the classical results in LQR by Anderson and Moore (see Anderson and Moore, 1971), a simple transformation of the design model from unstable to stable was made by choosing a cost that contains an exponential factor, hence overcoming the numerical ill-conditioning problem (see Wang 2001c). In order to guarantee closed-loop stability of the predictive control system, the original weight matrices in the cost function are adjusted when the exponential weight is used. An important by-product

of this transformation is the creation of a prescribed degree of stability in the MPC algorithms, both continuous-time and discrete-time. This prescribed degree of stability in conjunction with the use of orthonormal functions made the tuning of the predictive control system a relatively easy task.

The idea of creating a prescribed degree of stability has been previously used in receding horizon control (see Kwon and Han, 2005, Yoon and Clarke, 1993), and the key in the previous approaches relies on the use of an exponentially increasing weight in the cost function, along with the solution obtained analytically from a Riccati approach. The distinguishing point for the approaches in this book is that instead of increasing, an exponentially decreasing weight is used in the cost function to resolve the numerical problem, followed by adjustment of the constant weight matrices to recover and create more stability margins if required. Use of an exponentially decreasing weight is counter-intuitive from a closed-loop stability point of view, however, it makes sense when the numerical issue is of concern.

Practice

Predictive control using orthonormal basis functions has been successfully used in numerous applications, mainly by my previous undergraduate and postgraduate students. The majority of the applications were based on MATLAB real-time Workshop and SIMULINK xPC target. The MATLAB programs I wrote over the years are often directly translated into MATLAB C or SIMULINK programs for xPC target, then the applications followed through. The MATLAB programs are useful for practitioners as a start point, adopting them later for their own applications. The MATLAB programs are explained in a tutorial manner, step by step, so that the reader can understand how the algorithms are developed.

Presentation

The first version of this book had a different structure from the current version. The book had begun with continuous-time predictive control using orthonormal basis functions. The pre-conference predictive control workshop in the American Control Conference (ACC) 2007 made me change my mind about the structure of this book. When preparing for the workshop, I realized that a discrete-time system offers a natural setting for the development of predictive control, and the design framework is easier to understand when it is presented in discrete time. The discussion with Mr Oliver Jackson (Editor, Springer) in (ACC) 2007 also helped me think harder on how to deliver the complex issues in a manner as simple as possible. In the end, I chose to present the materials in a bottom-up manner to suit the background of a fourth-year undergraduate student. The best way to do so is to actually try the presentation on a class of fourth year students and then find out what the difficult issues are. This

is indeed what I have tried. The first three chapters were taught in the classroom, and the students provided the suggestions and feedback, which helped to set the level and pace of the presentation. The book is intended for readers who have completed or are about to complete four years engineering studies with some basic knowledge in state-space design. The textbooks for an introduction to state-space methods include Franklin *et al.* (1991), Kailath (1980), Bay (1999) and Goodwin *et al.* (2000). However, some of the basic knowledge will be reviewed in the relevant chapters, and the book is self-contained with MATLAB tutorials and numerous examples. The targeted readers are students, practitioners, instructors and researchers who wish to apply predictive control.

Book Structure

The structure of the book is illustrated by the block diagram as shown in Figure 0.1. There are ten chapters in this book, and both continuous-time and discrete-time predictive control systems are introduced and discussed (see Chapters 1 to 8). Discrete-time algorithms are introduced first because of their strong relevance to industrial applications and their natural settings in the development. Chapters 9 and 10 contain both continuous-time and discrete-time systems, from using special state-space realization to implementation of those control systems via MATLAB Real-time Workshop and SIMULINK xPC Target. Each chapter contains MATLAB tutorials, which illustrate how the algorithms can be used in simulation, computation and implementation, and a problem section for practice of the design.

Chapter 1 is for beginners, where we will use simple examples to show the principle of receding horizon control, which underpins predictive control. The solutions are limited to simple analytical solutions. In Chapter 1, we will also discuss implementation using observer and observability with several simple examples. In Chapter 2, the basic ideas about constrained control are introduced within the framework of receding horizon control. The key is to formulate the constrained control problem as a real-time optimization problem subject to inequality constraints. The solution to this problem relies on application of a quadratic programming procedure. Given that the majority of the readers may not have studied constrained optimization before, in this chapter, we will also discuss optimization with equality and inequality constraints. The discussion is followed by the introduction of Hildreth's quadratic programming procedure, which offers simplicity and reliability in real-time implementation.

In essence, the core technique in the design of discrete-time MPC is based on optimizing the future control trajectory subject to plant operational constraints. In the traditional predictive control, as demonstrated in Chapter 1 and 2, by assuming a finite control horizon N_c and prediction horizon N_p , the difference of the control signal $\Delta u(k)$ for $k = 0, 1, 2, \dots, N_c - 1$ is captured by the control vector ΔU , while the rest of the $\Delta u(k)$ for $k = N_c, N_c + 1, \dots, N_p$

is assumed to be zero. The idea in Chapter 3 is to generalize the traditional design procedure by introducing a set of discrete Laguerre functions into the design. This generalization will help us in re-formulating the predictive control problem and simplifying the solutions, in addition to providing a set of new performance-tuning parameters that can be readily understood by engineers. Furthermore, a long control horizon can be realized through the exponential nature of the Laguerre functions, hence without using a large number of parameters. Several MATLAB tutorials are presented in this chapter for the design of discrete-time predictive control systems, with or without constraints.

It is fair to say that the majority of industrial control systems require integral action. In this book, the design models are embedded with integrators to achieve this objective, which is similar to other classical predictive control systems. Because of the embedded integrators, the prediction horizon N_p as a design parameter plays an important role in a predictive control system. In Chapter 4, we demonstrate that if it is chosen too short, the closed-loop

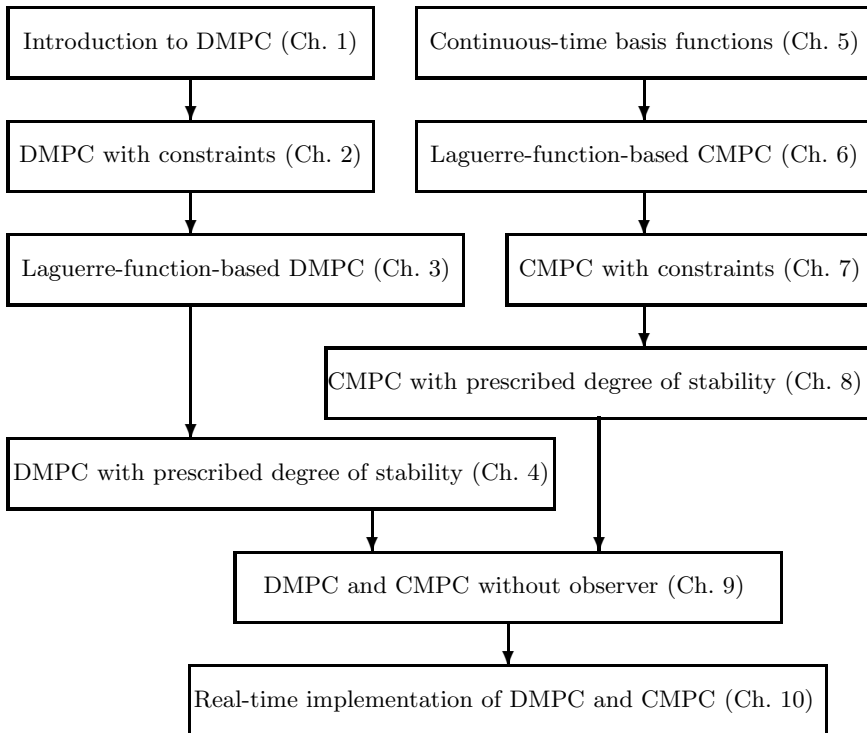


Fig. 0.1. Structure of the book

system is not necessarily stable, and if it is too large, the predictive control system will encounter a numerical stability problem. To overcome this problem, in Chapter 4, we propose the use of an exponentially weighted moving horizon window in model predictive control design. Within this framework, the numerical ill-conditioning problem is resolved by a simple modification of the design model. Equally important are an asymptotic stability result for predictive control designed using infinite horizon with exponential data weighting and a modification of the weight matrices, as well as a result that establishes the predictive control system with a prescribed degree of stability. Analytical and numerical results are used in this chapter to show the equivalence between the new class of discrete-time predictive control systems and the classical discrete-time linear quadratic regulators (DLQR) without constraints. When constraints are present, the optimal solutions are obtained by minimizing the exponentially weighted cost subject to transformed inequality constraints.

Chapters 6 to 8 will introduce the continuous-time predictive control results. To prepare the background, in Chapter 5, we will discuss continuous-time orthonormal basis functions and their applications in dynamic system modelling. Laguerre functions and Kautz functions are special classes of orthonormal basis functions. Both sets of functions possess simple Laplace transforms, and can be compactly represented by state-space models. The key property is that when using the orthonormal functions, modelling of the impulse response of a stable system, which has a bounded integral squared value, will have a guaranteed convergence as the number of terms used increases. This forms the fundamental principle of the model predictive control design methods presented in this book.

After introducing the background information, in Chapter 6, we begin with the topics in continuous-time model predictive control (CMPC). It is natural that when the design model is embedded with integrators, the derivative of the control signal should be modelled by the orthonormal basis functions, not the control signal itself. With this as a start point, systematically, we will cover the principles of continuous-time predictive control design, and the solutions of the optimal control problem. It shows that when constraints are not involved in the design, the continuous-time model predictive control scheme becomes a state feedback control system, with the gain being chosen from minimizing a finite prediction horizon cost. The continuous-time Laguerre functions and Kautz functions discussed in Chapter 5 are utilized in the design of continuous-time model predictive control.

In Chapter 7, we introduce continuous-time model predictive control with constraints. Similar to the discrete-time case, we will first formulate the constraints for the continuous-time predictive control system, and present the numerical solution of the constrained control problem using a quadratic programming procedure. Because of the nature of the continuous-time formulation such as fast sampling, there might be computational delays when a quadratic programming procedure is used in the solution of the real-time op-

timization problem. In general terms, we discuss the real-time implementation of continuous-time model predictive control in the presence of constraints in this chapter too.

The numerical instability problem discussed in the discrete-time case, also occurs in the continuous-time algorithms that is shown by a numerical example in Chapter 8. In a similar spirit to previous chapters, Chapter 8 proposes the use of exponential data weighting in the design of continuous-time model predictive control systems. This essentially transforms the original state and derivative of the control variables into exponentially weighted variables for the optimization procedure. With a simple modification on the weight matrices, asymptotic stability is established for model predictive control systems with infinite prediction horizon. Similarly, a prescribed degree of stability can also be obtained. Without constraints, analytical and numerical results are used to demonstrate the equivalence between the continuous-time model predictive controllers and the classical linear quadratic regulators (LQR). Constraints are imposed on the transformed variables.

In a general framework of state feedback control, an observer is often needed for its implementation. The design of an observer is a separate task from the design of predictive controller. The role of an observer is to ensure small errors between the estimated and the actual state variables. However, if one faces many inputs and many outputs in a system, tuning of an observer's dynamics may not be a trivial task. The classical predictive control systems, such as dynamic matrix control (DMC) and generalized predictive control (GPC), have directly utilized plant input and output signals in the closed-loop feedback control, hence avoiding observers in their implementation. In Chapter 9, we will link the predictive control systems designed using the framework of state space to the classical predictive control systems. The key to the link is to choose the state variables to be identical to the feedback variables that have been used in the classical predictive control systems. Once the state-space model is formulated, the framework from the previous chapters is naturally extended to the classical predictive control systems, preserving all the advantages of a state-space design, including stability analysis, exponential data weighting and LQR equivalence. In addition, because of the utilization of plant input and output signals in the implementation, the predictive controller can be represented in a transfer function form, allowing a direct frequency response analysis of the system to obtain critical information, such as gain and phase margins. It is worthwhile to point out that the discrete-time single-input and single-output predictive controller is very simple and easy for implementation. Simplicity as a feature of the algorithms remains when they are extended to multi-input and multi-output systems. Different from discrete-time, in the continuous-time design, an implementation filter is required, and the poles of the filter become part of the desired closed-loop poles when we choose to optimize the output errors in the design.

Chapter 10 presents three different implementation procedures for model predictive control systems. The first implementation is based on a (low-cost)

micro-controller for controlling a DC motor. In this application, the MATLAB design programs are utilized to calculate the predictive controller gain and the previous MATLAB closed-loop simulation program is converted to a C program for real-time implementation on the micro-controller. The second implementation is based on MATLAB Real-time Workshop and xPC target. This application is very useful for those who are not familiar with C language because the MATLAB Real-time Workshop and xPC target perform the conversion from MATLAB programs to C programs through their compilers in a systematic way. With these tools, we only need to create MATLAB embedded functions for the real-time applications. The third implementation uses the platform of a real-time PC-based supervisory control and data acquisition (SCADA) system. A pilot food extrusion plant is controlled by the continuous-time predictive controller developed in Chapter 6. The previous MATLAB closed-loop simulation program for a continuous-time system is converted to C program for this real-time implementation.

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There are two aspects that have motivated me in the writing of this book. The excitement of fitting together the work I have been doing over the years into a book is my first motivation. The second is that I need to express my gratitude to those who helped me over the years. With this book, I have the opportunity to formally thank so many colleagues and friends.

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Melbourne, Australia, October 2008

Liuping Wang

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List of Symbols and Abbreviations

Symbols

a	Scaling factor for discrete-time Laguerre functions
$\arg \min$	Minimizing argument
A	State matrix of state-space model
B	Input-to-state matrix of state-space model
C	State-to-output matrix of state-space model
D	Direct feed-through matrix of state-space model
(A, B, C, D)	State-space realization
ΔU	Parameter vector for the control sequence
$\Delta u(k_i + m)$	future incremental control at sample m
$\Delta u^{min}, \Delta u^{max}$	Minimum and maximum limits for Δu
F, Φ	Pair of matrices used in the prediction equation $Y = Fx(k_i) + \Phi\Delta U$
$G(s)$	Transfer function model
$I_{q \times q}$	Identity matrix with appropriate dimensions
J	Performance index for optimization
K_{lqr}	Feedback control gain using LQR
K_{mpc}	Feedback control gain using MPC
K_x	State feedback control gain vector related to $\Delta x_m(\cdot)$ or $\dot{x}_m(\cdot)$
K_y	State feedback control gain related to y
K_{ob}	Observer gain vector
$\kappa(A)$	Condition number of A matrix
$l_i(\cdot)$	The i th discrete or continuous-time Laguerre function
$L(\cdot)$	Discrete and continuous-time Laguerre functions in vector form
$L_i(s)$	Laplace transform of the i th continuous-time Laguerre function
$L_i(z)$	z-transform of the i th discrete Laguerre function
λ	Lagrange multiplier

$\lambda_i(A)$	The i th eigenvalue of matrix A
m	Number of inputs, also the m th future sample in discrete time
M, γ	Pair of matrix, vector for inequality constraints ($Mx \leq \gamma$)
N	Number of terms used in Laguerre function expansion, both continuous and discrete time
N_c	Control horizon
N_p	Prediction horizon
o_m	Zero vector with appropriate dimension
o_k	Zero row vector ($k = 1, 2, \dots$) with appropriate dimensions
$o_{q \times q}$	$q \times q$ zero matrix
$o_{q \times m}$	$q \times m$ zero matrix
Ω, Ψ	Pair of matrices in the cost of predictive control $J = \eta^T \Omega \eta + 2\eta^T \Psi x(\cdot) + cons$
η	Parameter vector in the Laguerre expansion
p	Scaling factor for continuous-time Laguerre functions
Q, R	Pair of weight matrices in the cost function of predictive control
q^{-i}	Backward shift operator, $q^{-i}[f(k)] = f(k - i)$
q	Number of outputs
$r(\cdot)$	Set-point signal
S_{act}	Index set of active constraints
T_p	Prediction horizon in continuous-time
$u(\cdot)$	Control signal
u^{min}, u^{max}	Minimum and maximum limits for u
$x(\cdot)$	State variable
$x(k_i + m k_i)$	Predicted state variable vector at sample time m , given current state $x(k_i)$
$x(t_i + \tau t_i)$	Predicted state variable vector at time τ given current state $x(t_i)$
$\hat{x}(\cdot)$	Estimated state variable vector in both continuous and discrete-time
$y(\cdot)$	Output signal
Y	Predicted output data vector
y^{min}, y^{max}	Minimum and maximum limits for y

Abbreviations

CMPC	Continuous-time model predictive control
DLQR	Discrete-time linear quadratic regulator

DMPC	Discrete-time model predictive control
FIR	Finite impulse response
LQR	Linear quadratic regulator
MIMO	Multiple-input, multiple-output
SISO	Single-input, single-output