Universitext

Achim Klenke

Probability Theory

A Comprehensive Course

Springer

Prof. Dr. Achim Klenke
Institut für Mathematik
Johannes Gutenberg-Universität Mainz
Staudingerweg 9
55099 Mainz
Germany

ISBN: 978-1-84800-047-6 e-ISBN: 978-1-84800-048-3
DOI: 10.1007/978-1-84800-048-3

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Control Number: 2007939558

Mathematics Subject Classification (2000): 60-01; 28-01

Translation from the German language edition:
Wahrscheinlichkeitstheorie by Achim Klenke

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

Springer Science+Business Media
springer.com

Preface

This book is based on two four-hour courses on advanced probability theory that I have held in recent years at the universities of Cologne and Mainz. It is implicitly assumed that the reader has a certain familiarity with the basic concepts of probability theory, although the formal framework will be fully developed in this book.

The aim of this book is to present the central objects and concepts of probability theory: random variables, independence, laws of large numbers and central limit theorems, martingales, exchangeability and infinite divisibility, Markov chains and Markov processes, as well as their connection with discrete potential theory, coupling, ergodic theory, Brownian motion and the Itô integral (including stochastic differential equations), the Poisson point process, percolation and the theory of large deviations.

Measure theory and integration are necessary prerequisites for a systematic probability theory. We develop it only to the point to which it is needed for our purposes: construction of measures and integrals, the Radon-Nikodym theorem and regular conditional distributions, convergence theorems for functions (Lebesgue) and measures (Prohorov) and construction of measures in product spaces. The chapters on measure theory do not come as a block at the beginning (although they are written such that this would be possible; that is, independent of the probabilistic chapters) but are rather interlaced with probabilistic chapters that are designed to display the power of the abstract concepts in the more intuitive world of probability theory. For example, we study percolation theory at the point where we barely have measures, random variables and independence; not even the integral is needed. As the only exception, the *systematic* construction of independent random variables is deferred to Chapter 14. Although it is rather a matter of taste, I hope that this setup helps to motivate the reader throughout the measure-theoretical chapters.

Those readers with a solid measure-theoretical education can skip in particular the first and fourth chapters and might wish only to look up this or that.

In the first eight chapters, we lay the foundations that will be needed in all subsequent chapters. After that, there are seven more or less independent parts, consisting of Chapters 9–12, 13, 14, 15–16, 17–19, 20 and 23. The chapter on Brownian motion (21) makes reference to Chapters 9–15. Again, after that, the three blocks consisting of Chapters 22, 24 and 25–26 can be read independently.

I should like to thank all those who read the manuscript and the German original version of this book and gave numerous hints for improvements: Roland Alkemper, René Billing, Dirk Brüggemann, Anne Eisenbürger, Patrick Jahn, Arnulf Jentzen, Ortwin Lorenz, L. Mayer, Mario Oeler, Marcus Schölpen, my colleagues Ehrhard Behrends, Wolfgang Bühler, Nina Gantert, Rudolf Grübel, Wolfgang König, Peter Mörters and Ralph Neininger, and in particular my colleague from Munich Hans-Otto Georgii. Dr John Preater did a great job language editing the English manuscript and also pointing out numerous mathematical flaws.

I am especially indebted to my wife Katrin for proofreading the English manuscript and for her patience and support.

I would be grateful for further suggestions, errors etc. to be sent by e-mail to

`math@aklenke.de`

Mainz,
October 2007

Achim Klenke

Contents