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Liapunov Functions and Stability in Control Theory

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Preface

We are interested in mathematical models of input systems, described by continuous-time, finite dimensional ordinary differential equations

$$\dot{x} = f(t, x, u) \tag{0.1}$$

where $t \geq 0$, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ represents the state variables, $u = (u_1, \ldots, u_m) \in \mathbb{R}^m$ represents the input variables and $f = (f_1, \ldots, f_n) : [0, +\infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. Together with (0.1), we will often consider the *unforced associated system*

$$\dot{x} = f(t, x, 0) \ . \tag{0.2}$$

Basically, (0.2) accounts for the "internal" behavior of the system. More precisely, (0.2) describes the natural dynamics of (0.1) when no energy is supplied through the input channels. The analysis of the "external" behavior is rather concerned with the effect of the inputs (disturbances or exogenous signals) on the evolution of the state response of (0.1).

Physical systems are usually expected to exhibit a "stable" behavior. A primary aim of this book is to survey some possible mathematical definitions of internal and external stability in a nonlinear context and to discuss their characterizations in the framework of the Liapunov functions method.

We will also consider the problem of achieving a more desirable stability behavior (both from the internal and the external point of view) by means of properly designed feedback laws. To this end, it is convenient to think of the input as a sum $u = u_e + u_c$. The term u_e represents external forces, while u_c is actually available for control action. Roughly speaking, (0.1) is said to be "stabilizable" if there exists a map $u_c = k(t, x)$ such that the closed loop system

$$\dot{x} = f(t, x, k(t, x) + u_e) \tag{0.3}$$

exhibits improved (internal and/or external) stability performances.

Intimate relationships among all these aspects of systems analysis emerge with some evidences from classical linear systems theory. In particular, as we shall see at the beginning of Chapter 2, the external behavior of a linear system is strongly related to its internal structure. On the contrary, dealing with nonlinear systems these connections become weaker and need a more delicate treatment.

We shall see in particular that the approach to stability and stabilizability of nonlinear systems rests much more heavily on the method of Liapunov functions. Thus, we are led to emphasize the interest in a variety of theorems which state, under minimal assumptions, the existence of Liapunov functions with suitable properties. These theorems are usually called "converse Liapunov theorems". A secondary aim of this book is to illustrate the state of the art on this subject, and to present some recent developments.

We have not yet specified what kind of assumptions should be made about the map f which appears at the right hand side of (0.1) and about the admissible inputs.

The class of admissible inputs should be so large to include representations of all signals commonly used in engineering applications. To this purpose, it is well known that in certain circumstances, a discontinuous function often is more suited than a continuous one. Thus, throughout these notes, we shall adopt the following agreement:

(I) the class of *admissible inputs* is constituted by all measurable, essentially bounded functions $u : [0, +\infty) \to \mathbb{R}^m$.

To establish the assumptions about f is a more delicate task. In a classical "smooth" setting, it seems natural to ask that f is time invariant, namely $f(t, x, u) = f(x, u)$, and at least continuous as a function of x, u, though additional regularity could be required for certain purposes[1]. This is actually the point of view we intend to adopt at the beginning but, as long as we proceed in our exposition, it will become clear that the smooth setting is too conservative for certain developments. This occurs in particular when we seek Liapunov functions of (Liapunov or Lagrange) stable systems or when we aim to design internally asymptotically stabilizing feedback laws. Indeed, the solution of

[1]Recent results of the so-called geometric control theory apply to systems whose right hand side can be represented as a family of C^∞ or real analytic vector fields (see [70], [71], [137]).

these problems cannot be found in general within a pre-assigned class of time invariant smooth functions, unless severe restrictions are made on the system under consideration. We will be so led to introduce in our treatment nondifferentiable functions and differential equations with discontinuous right hand side.

To be prepared for this extension, in Chapter 1 we recall some preliminary material about existence of solutions for ordinary differential equations and differential inclusions.

The main subject will be addressed starting from Chapter 2. As already mentioned, in Chapter 2 we focus more precisely on the case where the right hand side of (0.1) is time invariant and continuous with respect to both x, u. The reason why we prefer to begin with such a restricted class of systems is twofold. First, the more general approach could be felt at that point unmotivated and too abstract. Second, the main notions, methods and achievements available in the literature about stability and stabilizability theory of control systems have been mostly obtained, in the last few years, just for this class of systems. Of course, this choice implies also a few of complications (for instance, the need of a progressive updating of definitions and results when we shall undertake certain extensions) but gives a clearer perspective of problems and theoretical difficulties.

A first attempt to re-interpret our problems in a more general context is made in Chapter 3, where we consider time varying systems. We focus in particular on possible notions of internal stability and on their relationships. Although we are able to give some more precise results about existence of Liapunov functions and of stabilizing feedback, we shall see that the picture of the situation is not yet completely satisfactory.

The goal of replacing the classical smooth setting by a more general time dependent and "nonsmooth" one, will be fully pursued in Chapter 4, where we finally consider systems of the general form (0.1), and f is allowed to be discontinuous with respect to x. More precisely, in Chapter 4 we discuss direct and converse theorems about stability and asymptotic stability, together with their applications to external stabilization. We present also a new approach which allows us to prove in a unified manner several recent results. The proof given here is considerably shorter and easier than other proofs available in the original papers.

Certain additional properties of Liapunov functions will be discussed in Chapter 5. Here, we consider again the case of systems of ordinary differential equations, with time invariant and smooth right hand side. The topics include existence of analytic or homogeneous Liapunov functions and their symmetries, and relationship between Liapunov functions and decay of trajectories.

Finally, in Chapter 6 we review some tools from nonsmooth analysis which can be useful in the investigation of nondifferentiable systems with discontinuous Liapunov functions.

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