Advanced Textbooks in Control and Signal Processing
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Robotics

Modelling, Planning and Control
to our families
The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialised tutorial workshop or as a course to undergraduate, graduate or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

Robots have appeared extensively in the artistic field of science fiction writing. The actual name robot arose from its use by the playwright Karel Čapek in the play *Rossum's Universal Robots* (1920). Not surprisingly, the artistic focus has been on mechanical bipeds with anthropomorphic personalities often termed androids. This focus has been the theme of such cinematic productions as, *I, Robot* (based on Isaac Asimov’s stories) and Stanley Kubrick’s film, *A.I.*; however, this book demonstrates that robot technology is already widely used in industry and that there is some robot technology which is at prototype stage rapidly approaching introduction to commercial use. Currently, robots may be classified according to their mobility attributes as shown in the figure.
The largest class of robots extant today is that of the fixed robot which
does repetitive but often precise mechanical and physical tasks. These robots
pervade many areas of modern industrial automation and are mainly con-
cerned with tasks performed in a structured environment. It seems highly
likely that as the technology develops the number of mobile robots will signif-
ically increase and become far more visible as more applications and tasks
in an unstructured environment are serviced by robotic technology.

What then is robotics? A succinct definition is given in The Chamber’s Dic-
tionary (2003): the branch of technology dealing with the design, construction
and use of robots. This definition certainly captures the spirit of this volume
in the Advanced Textbooks in Control and Signal Processing series entitled
Robotics and written by Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani and
Giuseppe Oriolo. This book is a greatly extended and revised version of an
earlier book in the series, Modelling and Control of Robot Manipulators (2000,
ISBN: 978-1-85233-221-1). As can be seen from the figure above, robots cover
a wide variety of types and the new book seeks to present a unified approach
to robotics whilst focusing on the two leading classes of robots, the fixed and
the wheeled types. The textbook series publishes volumes in support of new
disciplines that are emerging with their own novel identity, and robotics as
a subject certainly falls into this category. The full scope of robotics lies at
the intersection of mechanics, electronics, signal processing, control engineer-
ing, computing and mathematical modelling. However, within this very broad
framework the authors have pursued the themes of modelling, planning and
control. These are, and will remain, fundamental aspects of robot design and
operation for years to come. Some interesting innovations in this text include
material on wheeled robots and on vision as used in the control of robots.
Thus, the book provides a thorough theoretical grounding in an area where
the technologies are evolving and developing in new applications.

The series is one of textbooks for advanced courses, and volumes in the
series have useful pedagogical features. This volume has twelve chapters cov-
ering both fundamental and specialist topics, and there is a Problems section
at the end of each chapter. Five appendices have been included to give more
depth to some of the advanced methods used in the text. There are over twelve
pages of references and nine pages of index. The details of the citations and
index should also facilitate the use of the volume as a source of reference as
well as a course study text. We expect that the student, the researcher, the
lecturer and the engineer will find this volume of great value for the study of
robotics.

Glasgow
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In the last 25 years, the field of robotics has stimulated an increasing interest in a wide number of scholars, and thus literature has been conspicuous, both in terms of textbooks and monographs, and in terms of specialized journals dedicated to robotics. This strong interest is also to be attributed to the interdisciplinary character of robotics, which is a science having roots in different areas. Cybernetics, mechanics, controls, computers, bioengineering, electronics — to mention the most important ones — are all cultural domains which undoubtedly have boosted the development of this science.

Despite robotics representing as yet a relatively young discipline, its foundations are to be considered well-assessed in the classical textbook literature. Among these, modelling, planning and control play a basic role, not only in the traditional context of industrial robotics, but also for the advanced scenarios of field and service robots, which have attracted an increasing interest from the research community in the last 15 years.

This book is the natural evolution of the previous text Modelling and Control of Robot Manipulators by the first two co-authors, published in 1995, and in 2000 with its second edition. The cut of the original textbook has been confirmed with the educational goal of blending the fundamental and technological aspects with those advanced aspects, on a uniform track as regards a rigorous formalism.

The fundamental and technological aspects are mainly concentrated in the first six chapters of the book and concern the theory of manipulator structures, including kinematics, statics and trajectory planning, and the technology of robot actuators, sensors and control units.

The advanced aspects are dealt with in the subsequent six chapters and concern dynamics and motion control of robot manipulators, interaction with the environment using exteroceptive sensory data (force and vision), mobile robots and motion planning.

The book contents are organized in 12 chapters and 5 appendices.

In Chap. 1, the differences between industrial and advanced applications are enlightened in the general robotics context. The most common mechanical
structures of robot manipulators and wheeled mobile robots are presented. Topics are also introduced which are developed in the subsequent chapters.

In Chap. 2 kinematics is presented with a systematic and general approach which refers to the Denavit-Hartenberg convention. The direct kinematics equation is formulated which relates joint space variables to operational space variables. This equation is utilized to find manipulator workspace as well as to derive a kinematic calibration technique. The inverse kinematics problem is also analyzed and closed-form solutions are found for typical manipulation structures.

Differential kinematics is presented in Chap. 3. The relationship between joint velocities and end-effector linear and angular velocities is described by the geometric Jacobian. The difference between the geometric Jacobian and the analytical Jacobian is pointed out. The Jacobian constitutes a fundamental tool to characterize a manipulator, since it allows the determination of singular configurations, an analysis of redundancy and the expression of the relationship between forces and moments applied to the end-effector and the resulting joint torques at equilibrium configurations (statics). Moreover, the Jacobian allows the formulation of inverse kinematics algorithms that solve the inverse kinematics problem even for manipulators not having a closed-form solution.

In Chap. 4, trajectory planning techniques are illustrated which deal with the computation of interpolating polynomials through a sequence of desired points. Both the case of point-to-point motion and that of motion through a sequence of points are treated. Techniques are developed for generating trajectories both in the joint space and in the operational space, with a special concern to orientation for the latter.

Chapter 5 is devoted to the presentation of actuators and sensors. After an illustration of the general features of an actuating system, methods to control electric and hydraulic drives are presented. The most common proprioceptive and exteroceptive sensors in robotics are described.

In Chap. 6, the functional architecture of a robot control system is illustrated. The characteristics of programming environments are presented with an emphasis on teaching-by-showing and robot-oriented programming. A general model for the hardware architecture of an industrial robot control system is finally discussed.

Chapter 7 deals with the derivation of manipulator dynamics, which plays a fundamental role in motion simulation, manipulation structure analysis and control algorithm synthesis. The dynamic model is obtained by explicitly taking into account the presence of actuators. Two approaches are considered, namely, one based on Lagrange formulation, and the other based on Newton–Euler formulation. The former is conceptually simpler and systematic, whereas the latter allows computation of a dynamic model in a recursive form. Notable properties of the dynamic model are presented, including linearity in the parameters which is utilized to develop a model identification technique. Finally,
the transformations needed to express the dynamic model in the operational space are illustrated.

In Chap. 8 the problem of motion control in free space is treated. The distinction between joint space decentralized and centralized control strategies is pointed out. With reference to the former, the independent joint control technique is presented which is typically used for industrial robot control. As a premise to centralized control, the computed torque feedforward control technique is introduced. Advanced schemes are then introduced including PD control with gravity compensation, inverse dynamics control, robust control, and adaptive control. Centralized techniques are extended to operational space control.

Force control of a manipulator in contact with the working environment is tackled in Chap. 9. The concepts of mechanical compliance and impedance are defined as a natural extension of operational space control schemes to the constrained motion case. Force control schemes are then presented which are obtained by the addition of an outer force feedback loop to a motion control scheme. The hybrid force/motion control strategy is finally presented with reference to the formulation of natural and artificial constraints describing an interaction task.

In Chap. 10, visual control is introduced which allows the use of information on the environment surrounding the robotic system. The problems of camera position and orientation estimate with respect to the objects in the scene are solved by resorting to both analytical and numerical techniques. After presenting the advantages to be gained with stereo vision and a suitable camera calibration, the two main visual control strategies are illustrated, namely in the operational space and in the image space, whose advantages can be effectively combined in the hybrid visual control scheme.

Wheeled mobile robots are dealt with in Chap. 11, which extends some modelling, planning and control aspects of the previous chapters. As far as modelling is concerned, it is worth distinguishing between the kinematic model, strongly characterized by the type of constraint imposed by wheel rolling, and the dynamic model which accounts for the forces acting on the robot. The peculiar structure of the kinematic model is keenly exploited to develop both path and trajectory planning techniques. The control problem is tackled with reference to two main motion tasks: trajectory tracking and configuration regulation. Further, it is evidenced how the implementation of the control schemes utilizes odometric localization methods.

Chapter 12 reprises the planning problems treated in Chaps. 4 and 11 for robot manipulators and mobile robots respectively, in the case when obstacles are present in the workspace. In this framework, motion planning is referred to, which is effectively formulated in the configuration space. Several planning techniques for mobile robots are then presented: retraction, cell decomposition, probabilistic, artificial potential. The extension to the case of robot manipulators is finally discussed.
This chapter concludes the presentation of the topical contents of the textbook; five appendices follow which have been included to recall background methodologies.

Appendix A is devoted to linear algebra and presents the fundamental notions on matrices, vectors and related operations.

Appendix B presents those basic concepts of rigid body mechanics which are preliminary to the study of manipulator kinematics, statics and dynamics.

Appendix C illustrates the principles of feedback control of linear systems and presents a general method based on Lyapunov theory for control of nonlinear systems.

Appendix D deals with some concepts of differential geometry needed for control of mechanical systems subject to nonholonomic constraints.

Appendix E is focused on graph search algorithms and their complexity in view of application to motion planning methods.

The organization of the contents according to the above illustrated scheme allows the adoption of the book as a reference text for a senior undergraduate or graduate course in automation, computer, electrical, electronics, or mechanical engineering with strong robotics content.

From a pedagogical viewpoint, the various topics are presented in an instrumental manner and are developed with a gradually increasing level of difficulty. Problems are raised and proper tools are established to find engineering-oriented solutions. Each chapter is introduced by a brief preamble providing the rationale and the objectives of the subject matter. The topics needed for a proficient study of the text are presented in the five appendices, whose purpose is to provide students of different extraction with a homogeneous background.

The book contains more than 310 illustrations and more than 60 worked-out examples and case studies spread throughout the text with frequent resort to simulation. The results of computer implementations of inverse kinematics algorithms, trajectory planning techniques, inverse dynamics computation, motion, force and visual control algorithms for robot manipulators, and motion control for mobile robots are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of application in practical problems. In addition, nearly 150 end-of-chapter problems are proposed, some of which contain further study matter of the contents, and the book is accompanied by an electronic solutions manual (downloadable from www.springer.com/978-1-84628-641-4) containing the MATLAB® code for computer problems; this is available free of charge to those adopting this volume as a text for courses. Special care has been devoted to the selection of bibliographical references (more than 250) which are cited at the end of each chapter in relation to the historical development of the field.

Finally, the authors wish to acknowledge all those who have been helpful in the preparation of this book.

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