

Lecture Notes  
in Control and Information Sciences 269

---

Editors: M. Thoma · M. Morari

**Springer**

*Berlin*

*Heidelberg*

*New York*

*Barcelona*

*Hong Kong*

*London*

*Milan*

*Paris*

*Singapore*

*Tokyo*

**Engineering**  **ONLINE LIBRARY**

<http://www.springer.de/engine/>

Silviu-Iulian Niculescu

---

# **Delay Effects on Stability**

**A Robust Control Approach**

## Series Advisory Board

A. Bensoussan · P. Fleming · M.J. Grimble · P. Kokotovic ·  
A.B. Kurzhanski · H. Kwakernaak · J.L. Massey

## Author

Dr. Silviu-Iulian Niculescu  
Université de Technologie de Compiègne  
Centre de Recherche de Royallieu  
BP 20529  
F-60205 Compiègne, Cedex

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme  
Niculescu, Silviu-Iulian  
Delay effects on stability: a robust control approach / Silviu-Iulian Niculescu  
Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milano; Paris; Singapur; Tokyo:  
Springer, 2001  
(Lecture notes in control and information sciences; 269)  
(Engineering online library)  
ISBN 1-85233-291-3

ISBN 1-85233-291-3 Springer-Verlag London Berlin Heidelberg

Apart from any fair dealing for the purposes of research of private study, or criticism or review, as permitted under the Copyright, Design and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms of licence issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms should be sent to the publishers.

© Springer-Verlag London Limited 2001  
Printed in Germany

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant laws and regulation and therefore free for general use.

The publisher makes no representation, express or implied, with regard to the accuracy of the information contained in this book and cannot accept any legal responsibility or liability for any errors or omissions that may be made.

Typesetting: Digital data supplied by author, Data-Conversion by PTP-Berlin, Stefan Sossna  
Cover-Design: design & production GmbH, Heidelberg  
Printed on acid-free paper SPIN: 10746179 62/3020Rw - 5 4 3 2 1 0

# Preface

Most of the reactions of real (engineering) systems to external actions and signals *never* take place *instantaneously*. The same property holds for interconnected real systems or processes where (material, energy or information) transfer may occur due to some physical or chemical laws.

In other cases (biological or population dynamics), the assumption that the *future state* of the system is independent of the past states and depends *only* on *present* seems to be *insufficient* for describing the evolution of the system.

One of the ways to overcome such problems is to include in the mathematical model of the system dynamics *some* information on the *past* (states). Such systems are generically called *delay* systems, and they are infinite-dimensional.

This monograph is devoted to the *effect* of *delays* on the stability properties of dynamical systems. Stability regions with respect to the delay parameters are considered, and some sufficient (or necessary and sufficient) characterization are proposed. Note that if the stability characterization in the finite-dimensional linear case is completely known, the *problem* is still *open* for general (linear) delay systems. This monograph addresses such problem and gives solutions in some cases. For the others, approximations of the stability regions can be proposed, with some degree of conservativeness. The interpretation of *delays* as *uncertainty* allows us to use the advances in *robust control* (analysis and design) and *robust convex optimization* to compute or to approximate the exact solutions of the corresponding problems.

The monograph is organized as follows:

Motivation and an uncertainty interpretation of delays are outlined in the *Introduction*. Basic notions (definition of the state, existence of solutions, uniqueness, continuous dependence, stability definitions, etc.) are briefly presented. An overview of some topics and a historical perspective in the “delay field” (mathematics, control engineering) are also included.

Some (engineering and biology) *examples* from the literature are briefly presented in the second chapter. Most of them are reconsidered in the next chapters, where various techniques and methods are explicitly applied. Further examples can be found in the monographs of Kolmanovskii and Myshkis (1992), Kolmanovskii and Nosov (1986), Stépán (1989), Räsvan (1977), Gopal-samy (1992), MacDonald (1989), Kuang (1993), to cite only a few (see the list of references).

The third chapter is devoted to basic *delay effects* notions in *delay systems* and *propagation models*. The notions of *delay-independent* and *delay-dependent* are introduced and illustrated in the *scalar case* (retarded, neutral, propagation or integral delay equations). *Model transformations* and their stability conservativeness are largely treated.

The fourth chapter addresses the frequency-domain methods for the analysis of dynamical systems including discrete, or some special classes of dis-

tributed delays. Thus, *frequency-sweeping tests* or *Lagrange multipliers* are used in combination with appropriate *parametrized model transformations*. The delay elements are interpreted as *special fictitious inputs* for appropriate finite-dimensional linear time-invariant (LTI) systems. Various control interpretations are proposed. A special attention is paid to *matrix pencil* techniques, since the corresponding results are *necessary and sufficient conditions* for some special class of delay systems (point and commensurate delays). Comments on the conservatism of all the methods are also proposed. Various examples (from neural networks to active displacement and integrodifferential models for commodity markets) are treated. Analysis control techniques from the Tsytkin's (closed-loop) delay-independent stability criterion in the forties till the two variables (delay-independent) stability result of Kamen in the eighties are briefly presented.

The fifth chapter is devoted to the *Liapunov's second method*, and their applications. The advantages as well as the inconvenients of the Krasovskii functionals or Liapunov functions (Razumikhin, or Barnea's idea) are considered. A special attention is paid to the construction of Liapunov candidates combined with linear matrix inequalities (LMIs) (using special quadratic forms on appropriate space products), *discretization* (using complete quadratic candidates) or *control techniques* (feedforward/backstepping). Some examples (chemical engineering or neutral systems encountered in delay measurements) end the chapter.

The sixth chapter deals with some *robustness issues in closed-loop schemes*. Thus, the Smith controller and its robustness with respect to time-delay *uncertainty* is considered. Generalized Popov theory and passivity techniques are extended to the delay case. In the first case, sufficient conditions are derived using *matrix pencils*. In the second case, the analysis is reduced to the existence of solutions to appropriate algebraic Riccati equations (ARE). Note that most of the results proposed in the previous chapters can be extended to the design of *memoryless controllers*, such that the closed-loop systems satisfy appropriate properties. Closed-loop stability of linear systems subject to *delay inputs* are considered using frequency (integral quadratic constraints, IQCs) - and time-domain (Liapunov-Razumikhin) techniques.

The last chapter is devoted to some *applications*. Low-order (second or third) combustion models with discrete delays are treated. A complete characterization between the gains of the controllers and the delay values guaranteeing closed-loop stability is given in a frequency-domain approach. The adaptive Smith controller for strictly positive-real (SPR) systems is also proposed using appropriate Liapunov-Krasovskii functionals. The effects of delays on autonomous cruise control are analyzed in a frequency-domain setting. The *delay effects* on mechanical systems subject to unilateral constraints (manipulators in contact with rigid environments) are also considered. Two control laws (proportional or proportional-integral) are treated in detail. Special attention is paid to the delay output control problem related to the stabilization of oscillating systems. Some useful appendices are included.

The content of the monograph was substantially improved after discussions

and scientific collaboration with Prof. VLADIMIR B. KOLMANOVSKII (MIEM, Moscow, Russia) Prof. VLAD IONESCU (University ‘Politehnica’ of Bucharest, Romania), Prof. VLADIMIR RĂSVAN (University of Craiova, Romania) and Prof. ANURADHA M. ANNASWAMY (MIT Cambridge, United States). Some of the problems treated in the book were presented at the French and International workshops organized by the *GDR CNRS* group “*Analysis and synthesis of time-delay systems*” in the period 1994-1999 at Paris, Nantes or Grenoble. I wish to thank all the members of the group for their useful comments and discussions.

A *NSF-CNRS* project (1999-2002) devoted to *Delays effects* and including several French and American teams also contributed to important developments and collaborations related to *model transformations* and *output feedback problems* in delay systems. Special thanks go to Prof. KEQIN GU (University of Southern Illinois at Edwardsville, US), Prof. JIM LOUISELL (University of Southern Colorado at Pueblo, US), Prof. CHAOUKI T. ABDALLAH (University of New Mexico at Albuquerque, US) and Prof. ERIK I. VERRIEST (GeorgiaTech at Atlanta, US) for useful discussions and collaboration. Private discussions and communications with Prof. JIE CHEN (University of California at Riverside, US) opened some new and interesting research directions on frequency-sweeping tests and (parametrized) model transformations. A CNRS-Academy of Sciences (Bucharest, Romaina) project (2000) devoted to *Optimization in dynamical systems* contributed to important developments on the extension of Popov theory to delay systems (including some special cases of convolution operators).

Parts of the monograph were developed during the author’s stay at the Department of Applied Mathematics, ENSTA, Paris (France), 1996-1997. Fruitful discussions with Dr. LAURENT EL GHAOUI (now at University of California at Berkeley, US) and Dr. HUGO WOERDEMAN (from William & Mary College, US) on numerical algorithms (in convex optimization) contributed to the improvement of some of the results presented. I would also wish to thank Dr. BERNARD BROGLIATO from Laboratoire d’Automatique de Grenoble (France) and Dr. ROGELIO LOZANO from HeuDiaSyC (Compiègne, France) for their friendship and encouragements, and our long scientific collaboration. Special thanks go to Dr. HUAIZHONG LI (University of Perth, Australia), Prof. MINYUE FU (University of Newcastle, Australia) and Prof. CARLOS E. DE SOUZA (LNCC, Petropolis, Brazil) for our scientific collaboration and fruitful discussions during their visits in France (Grenoble and/or Compiègne).

Note also that some chapters of the monograph were used as *Course Notes* in the *Nonlinear systems* course at the *M.Sc. ‘Contrôle des Systèmes’* (since 1997) at HeuDiaSyC, UTCompiègne, and the ‘student feedback’ (remarks, questions, discussions) simplified the presentation of the material.

Furthermore, a *Summer school* devoted to *delay systems: Analysis and control of delay systems* that was held in Grenoble in September 2000 was at the origin and motivated *several examples* in the second chapter. In this sense, the low order *combustion* systems and their delay-induced instabilities were developed in collaboration with A. M. ANNASWAMY (a NSF-CNRS cooperation with R. LOZANO from HeuDiaSyC), and delay effects in *propagation models* with VL. RĂSVAN during his stay at HeuDiaSyC and at UTCompiègne in 1999

and 2000. A synthesis of the corresponding results can be found in: *Systems with propagation: Analysis and control* (monograph in preparation). The effect of delay measurements on manipulators subject to unilateral constraints was developed in collaboration with B. BROGLIATO, and the passivity approach related to some bilateral control of telemanipulators was developed with R. LOZANO. Furthermore, the Smith adaptive controller in the last chapter was developed in collaboration with A. M. ANNASWAMY.

I am grateful to my friends: PIERRE-ALEXANDRE BLIMAN, FRÉDÉRIC MAZENC (both from INRIA, France) for long discussions on nonlinear systems and applications. Special thanks go to my Ph.D. advisors: Dr. JEAN-MICHEL DION, and Dr. LUC DUGARD from Laboratoire d'Automatique de Grenoble for their support, encouragements and our scientific collaboration during the last eight years.

Note that this monograph completes and improves the results proposed by the author in the French monograph: *Delay systems. Qualitative aspects on the stability and stabilization* (Paris, October 1997), which was an “extended version” of the Ph.D. thesis (Grenoble, February 1996). At last but not the least, I have to mention the beneficial influence of a “huge name” in Mathematics, Prof. ARISTIDE HALANAY, who indirectly helped me in defining research directions in my scientific career. I shall never forget the scientific seminars at University of Bucharest (Romania), Department of Mathematics organized by himself with V. IONESCU and VL. RĂSVAN.

Special thanks are addressed to Prof. EDUARDO SONTAG (Rutgers University, US) who considered the subject of interest to be published. Finally, I am grateful to HANNAH RANSLEY and to NICHOLAS PINFIELD from SPRINGER-VERLAG, London for their help and their patience during the preparation of the camera-ready copy. Reading the books: *A history of reading* (A. Manguel; English edition at Harper Collins: London, 1986) and *Handbook of writing for mathematical sciences* (N. J. Higham; SIAM: Philadelphia, 1993) was a pleasure and helped me during the preparation of the manuscript.

There is a special person in my life, LAURA, to whom I owe the exceptional support that she gave to me to overcome all the difficulties encountered both professional and extra-professional. She influenced me to go on with the research work during all these years. I dedicate this monograph to her, in love and gratitude.

Compiègne, November 2000

SILVIU-IULIAN NICULESCU



# Contents

<b>Notations and acronyms</b>	<b>xv</b>
<b>1 Preliminaries</b>	<b>1</b>
1.1 Problem statement . . . . .	3
1.1.1 System class . . . . .	3
1.1.2 Delay effects on stability: open problem . . . . .	5
1.1.3 Delay: an uncertainty interpretation . . . . .	6
1.2 Models: representations and transformations . . . . .	8
1.2.1 System representations . . . . .	9
1.2.2 On the transformations . . . . .	11
1.2.3 Discrete/distributed delays . . . . .	14
1.2.4 Propagation models . . . . .	16
1.3 Basic results in RFDE . . . . .	20
1.3.1 The notion of state . . . . .	20
1.3.2 Solutions: definitions, existence, uniqueness . . . . .	21
1.3.3 Step method and smoothness . . . . .	22
1.4 Stability in RFDE . . . . .	24
1.4.1 Basic definitions . . . . .	24
1.4.2 Characteristic equations . . . . .	25
1.4.3 Some analytical and graphical tests . . . . .	27
1.4.4 Liapunov's second method . . . . .	30
1.5 Basic notions and stability in NFDE . . . . .	33
1.5.1 Basic results . . . . .	33
1.5.2 Characteristic equations . . . . .	34
1.5.3 Liapunov's second method . . . . .	35
1.6 Basic results in lossless propagation models . . . . .	36
1.6.1 Some definitions . . . . .	37
1.6.2 Characteristic equation . . . . .	38
1.6.3 Connections with the solutions of hyperbolic PDE . . . . .	38
1.6.4 Connections with neutral systems . . . . .	39
1.6.5 Step method . . . . .	40
1.6.6 Some remarks on IDE . . . . .	40
1.7 Special topics: degenerate Liapunov . . . . .	41
1.7.1 More definitions . . . . .	41

1.7.2	Stability result . . . . .	42
1.8	Notes and comments . . . . .	43
1.8.1	Delay equations: a brief history . . . . .	44
1.8.2	About the monograph . . . . .	46
<b>2</b>	<b>Examples</b>	<b>47</b>
2.1	Transport and communication delays . . . . .	48
2.1.1	Chemical engineering . . . . .	48
2.1.2	Combustion models . . . . .	51
2.1.3	Control strategy in vehicle following systems . . . . .	54
2.1.4	Telemanipulation systems . . . . .	57
2.1.5	Congestion avoidance in high-speed internet . . . . .	60
2.1.6	Neural Networks . . . . .	63
2.2	Delay measurements . . . . .	66
2.2.1	Active displacement control (flexible structure) . . . . .	66
2.2.2	Robots in contact with rigid environments . . . . .	68
2.3	Heredity: biology and population dynamics . . . . .	71
2.3.1	Simplified population growth model . . . . .	71
2.3.2	Interconnection structure: a source of models . . . . .	72
2.3.3	Some pulse circulation models . . . . .	74
2.4	Dynamics: Reducing / Inducing delays . . . . .	76
2.4.1	Smith principle: reducing delays . . . . .	76
2.4.2	Oscillations and delayed output . . . . .	78
2.4.3	Delayed feedback: chaotic and helicopter dynamics . . . . .	80
2.5	Propagation phenomena . . . . .	83
2.5.1	Electrical-circuit models . . . . .	83
2.5.2	Hydraulic engineering models . . . . .	85
<b>3</b>	<b>Stability sets and regions</b>	<b>87</b>
3.1	Definitions and basic ideas . . . . .	87
3.1.1	Discrete delays: definitions and related remarks . . . . .	87
3.1.2	Extensions to time-varying delays . . . . .	95
3.1.3	More general distributed delays . . . . .	96
3.1.4	On neutral systems and lossless propagation . . . . .	98
3.2	Model transformations: retarded case . . . . .	100
3.2.1	Fixed first-order transformation . . . . .	100
3.2.2	Fixed second-order transformations . . . . .	103
3.2.3	Neutral transformations . . . . .	106
3.2.4	Parametrized first-order transformations . . . . .	107
3.2.5	Control interpretations . . . . .	111
3.3	Model transformations: neutral case . . . . .	112
3.3.1	Fixed first-order transformations . . . . .	112
3.3.2	Neutral transformations . . . . .	113
3.3.3	Parametrized first-order transformations . . . . .	115
3.3.4	On lossless propagation model transformations . . . . .	116
3.4	Scalar systems . . . . .	118

3.4.1	Retarded case . . . . .	119
3.4.2	Neutral case . . . . .	124
3.5	Analysis control-based techniques . . . . .	127
3.5.1	Frequency-domain: reducing techniques . . . . .	127
3.5.2	Liapunov method: LMI solutions . . . . .	129
<b>4</b>	<b>Reducible discrete delays and LTIs</b>	<b>131</b>
4.1	Introductory remarks . . . . .	131
4.1.1	Tsytkin and frequency-sweeping . . . . .	132
4.1.2	Kamen and multivariable polynomials . . . . .	133
4.1.3	Pseudo-delay technique and quasipolynomials . . . . .	136
4.1.4	Some necessary delay-independent conditions . . . . .	138
4.2	Lagrange multipliers . . . . .	140
4.2.1	Model transformations . . . . .	140
4.2.2	Delay transformations and approximations . . . . .	142
4.2.3	Constructing scalings . . . . .	145
4.3	Well-posedness of associated systems . . . . .	147
4.3.1	Defining interconnection schemes . . . . .	149
4.3.2	Frequency-sweeping tests . . . . .	150
4.3.3	Various criteria and related remarks . . . . .	160
4.3.4	Spectral radius and control interpretations . . . . .	166
4.3.5	Still maximum principle idea . . . . .	167
4.3.6	On the complexity issues . . . . .	168
4.4	Matrix pencils techniques . . . . .	169
4.4.1	Delay-independent criteria . . . . .	169
4.4.2	Delay-dependent criteria: first delay interval . . . . .	173
4.4.3	Delay switches: general delay intervals . . . . .	174
4.4.4	Hyperbolicity . . . . .	177
4.4.5	Related remarks in the general case . . . . .	180
4.5	Delay reduction in lossless propagation . . . . .	180
4.5.1	Frequency-sweeping tests . . . . .	181
4.5.2	Control interpretations . . . . .	184
4.5.3	Further comments . . . . .	186
4.6	Some Examples . . . . .	187
4.6.1	Controlling simple delay systems . . . . .	187
4.6.2	A linearized neural network model . . . . .	189
4.6.3	Delay measurements in active displacement . . . . .	190
4.6.4	Integro-differential models for commodity markets . . . . .	192
4.6.5	Delay circuits analysis in VLSI systems . . . . .	193
4.6.6	Lossless propagation . . . . .	193
<b>5</b>	<b>Liapunov's second method and LMIs</b>	<b>197</b>
5.1	Simple quadratic Liapunov candidates . . . . .	198
5.1.1	Scalar case . . . . .	198
5.1.2	Liapunov-Razumikhin functions . . . . .	205
5.1.3	Liapunov-Krasovskii functionals . . . . .	207

5.2	Complete quadratic Liapunov candidates . . . . .	209
5.3	Constructing and interpreting Liapunov . . . . .	212
5.3.1	An energy-based construction . . . . .	213
5.3.2	Control Liapunov functions . . . . .	216
5.3.3	Generalized Popov theory interpretations . . . . .	218
5.3.4	Liapunov and frequency-domain interpretations . . . . .	219
5.3.5	Model transformations and discretization . . . . .	220
5.4	Model transformations in retarded systems . . . . .	220
5.4.1	Neutral model transformations . . . . .	221
5.4.2	Parametrized model transformation . . . . .	223
5.5	Model transformation in neutral systems . . . . .	227
5.5.1	Neutral transformations . . . . .	228
5.5.2	Parametrized model transformations . . . . .	233
5.6	Simple distributed delay systems . . . . .	236
5.6.1	On some model transformations . . . . .	236
5.6.2	Discretization of Liapunov functionals . . . . .	237
5.7	On comparison principle . . . . .	245
5.7.1	Matrix measures: constant delays . . . . .	245
5.7.2	Matrix measures: time-varying delays . . . . .	246
5.7.3	Other measures: $M$ -matrices . . . . .	248
5.8	Examples . . . . .	248
5.8.1	Transport delay in chemical reactions . . . . .	249
5.8.2	A simple model with delay in force feedback . . . . .	250
5.8.3	On some nuclear reactors models . . . . .	251
<b>6</b>	<b>Robustness issues in closed-loop</b> . . . . .	<b>253</b>
6.1	General ideas . . . . .	253
6.1.1	Memoryless controllers . . . . .	254
6.1.2	Input delays . . . . .	254
6.1.3	Notes and comments . . . . .	255
6.2	Delay robustness of Smith controllers . . . . .	255
6.2.1	Problem formulation . . . . .	255
6.2.2	Delay effects . . . . .	256
6.3	Closed-loop stability of delay input systems . . . . .	259
6.3.1	Problem formulation . . . . .	259
6.3.2	Razumikhin approach . . . . .	261
6.3.3	IQC approach . . . . .	261
6.3.4	Artstein's model reduction . . . . .	263
6.4	Generalized Popov theory in delay systems . . . . .	265
6.4.1	Discrete delays case . . . . .	265
6.4.2	Extensions to convolution equations . . . . .	270
6.5	Passivity of delay systems . . . . .	277
6.5.1	Pointwise delay case . . . . .	278
6.5.2	Extensions to some convolution operators . . . . .	280

<b>7 Applications</b>	<b>283</b>
7.1 Delay effects in combustion models stability . . . . .	283
7.1.1 Associated low-order systems . . . . .	283
7.1.2 Second-order system . . . . .	285
7.1.3 Third-order system . . . . .	289
7.2 Delay effects in autonomous cruise control . . . . .	294
7.2.1 Problem formulation . . . . .	295
7.2.2 Individual vehicle stability . . . . .	295
7.2.3 Avoiding slinky effects . . . . .	297
7.3 Simple time-delay adaptive controllers . . . . .	298
7.3.1 Problem formulation and special cases . . . . .	299
7.3.2 Model transformation and stability analysis . . . . .	305
7.3.3 Further comments . . . . .	308
7.3.4 Extensions to the case when all states are accessible . . . . .	310
7.4 Output feedback in presence of delays . . . . .	312
7.4.1 Problem formulation . . . . .	313
7.4.2 Existence Results . . . . .	314
7.4.3 Constructing algorithms: a case study . . . . .	316
7.4.4 Stabilizing oscillations by delayed output . . . . .	317
7.5 Contact Instability Phenomenon . . . . .	318
7.5.1 Problem formulation . . . . .	318
7.5.2 Conditions for interaction force constant negative sign . . . . .	319
7.5.3 Bouncing phase analysis . . . . .	321
7.5.4 Further remarks . . . . .	325
<b>A Various definitions</b>	<b>327</b>
A.1 Matrix measures and $M$ -matrices . . . . .	327
A.1.1 Matrix measures: definitions and properties . . . . .	327
A.1.2 $M$ -matrices: definitions and properties . . . . .	328
A.2 $\mu$ -analysis . . . . .	328
A.2.1 Structured singular value . . . . .	328
A.2.2 Simple properties . . . . .	329
A.3 On the complexity of decision problems . . . . .	330
A.3.1 $\mathcal{P}$ and $\mathcal{NP}$ problems . . . . .	330
A.3.2 $\mathcal{NP}$ -completeness, $\mathcal{NP}$ -hardness . . . . .	330
A.4 Matrix pencils . . . . .	330
A.4.1 Definitions . . . . .	330
A.4.2 Dichotomy . . . . .	331
A.5 Passivity theory . . . . .	331
A.5.1 Passive systems . . . . .	331
A.5.2 Strictly passive systems . . . . .	331
<b>B Useful lemmas</b>	<b>333</b>
B.1 Barbălat lemma . . . . .	333
B.2 KYP lemma . . . . .	333
B.3 IQC filter construction . . . . .	334

<b>C</b>	<b>Computational aspects</b>	<b>335</b>
C.1	Kronecker sums and products . . . . .	335
C.1.1	Definitions . . . . .	335
C.1.2	Basic properties . . . . .	335
C.1.3	More general tensor products and sums . . . . .	337
C.2	Linear Matrix Inequality (LMI) . . . . .	338
C.2.1	Definition . . . . .	338
C.2.2	Optimization problems . . . . .	338
C.2.3	$\mathcal{S}$ -procedure . . . . .	339
C.2.4	Elimination lemma . . . . .	339
<b>D</b>	<b>Generalized Popov theory</b>	<b>341</b>
D.1	Popov “objects” . . . . .	341
D.1.1	Popov triplets . . . . .	341
D.1.2	KYP system in $J$ form . . . . .	341
D.1.3	Stabilizing solutions . . . . .	342
D.1.4	Disconjugacy . . . . .	342
D.2	Basic results . . . . .	342
D.2.1	Some matrix pencil characterizations . . . . .	342
D.2.2	Some remarks on Lur’e systems . . . . .	343
	<b>Bibliography</b>	<b>345</b>
	<b>Index</b>	<b>381</b>

# Notations and acronyms

## Notations

$\mathbb{R}$  ( $\mathbb{C}$ ) denotes the set of real (complex) numbers;

$\mathbb{R}^*$  denotes  $\mathbb{R} - \{0\}$  ( $\mathbb{C}^* = \mathbb{C} - \{0\}$ );

$\mathcal{C}(0,1)$  denotes the unit circle in the complex plane;

$z \in \mathbb{C}$ ,  $\bar{z}$  denotes its complex conjugate;

$\mathbb{R}^+$  is the set of non-negative real numbers;

$j\mathbb{R}$  denotes the imaginary axis of the complex plane ( $j\mathbb{R}^* = j\mathbb{R} - \{0\}$ );

$\mathbb{R}^n$  denotes the  $n$  dimensional Euclidean space;

$\mathbb{R}^{n \times m}$  ( $\mathbb{C}^{n \times m}$ ) denotes the set of all  $n \times m$  real (complex) matrices;

$\Lambda(M)$  represents the set of eigenvalues (spectrum) of the matrix  $M \in \mathbb{C}^{n \times n}$ ;

$\text{diag}(A, B)$  denotes the matrix  $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ , where the zero blocks have appropriate dimensions for the matrices  $A \in \mathbb{C}^{m_1 \times n_1}$ ,  $B \in \mathbb{C}^{m_2 \times n_2}$ ;

$\mathbb{C}^+$  ( $\mathbb{C}^-$ ) denotes the open right (left) half complex plane.

$\text{In}(M)$  is the inertia of the complex matrix  $M \in \mathbb{C}^{n \times n}$  and is equal to  $(\pi(M), \nu(M), \delta(M))$ , where  $\pi(M)$ ,  $\nu(M)$  and  $\delta(M)$  denote the number of eigenvalues with negative ( $\mathbb{C}^-$ ), positive ( $\mathbb{C}^+$ ) and zero real parts ( $j\mathbb{R}$ );

$\text{sgn}(f)$  denotes the sign of  $f$ ;

$\otimes, \oplus (p_{\otimes}, p_{\oplus})$  denotes the Kronecker product, sum (tensor product, sum);

$\mu(A)$  with  $A \in \mathbb{R}^{n \times n}$  denotes the matrix measure of  $A$  given by:

$$\mu(A) = \lim_{h \rightarrow 0^+} \frac{\|I_n + hA\| - 1}{h};$$

$\mathcal{C}_{n,\tau}$  or  $\mathcal{C}([-\tau, 0], \mathbb{R}^n)$  denotes the Banach space of continuous vector functions mapping the interval  $[-\tau, 0]$  into  $\mathbb{R}^n$  with the topology of uniform convergence.

The following *norms* will be used:

$\|\cdot\|$  refers to the Euclidean vector norm;

$\|\phi\|_c = \sup_{-\tau \leq t \leq 0} \|\phi(t)\|$  stands for the norm of a function  $\phi \in \mathcal{C}_{n,\tau}$ .

Moreover, we denote by  $\mathcal{C}_{n,\tau}^v$  the set defined by  $\mathcal{C}_{n,\tau}^v = \{\phi \in \mathcal{C}_{n,\tau} : \|\phi\|_c < v\}$ , where  $v$  is a positive real number.

## Acronyms

ABR:	available bit rate
ATM:	asynchronous transfer mode
ARE:	algebraic Riccati equation
DOF:	degree-of-freedom
EHP:	extended Hamiltonian pencil
EVP (GEVP):	eigenvalue optimization problem (generalized EVP)
FDE:	functional differential equation
FIFO:	first-in, first-out
IDE:	integral delay equations
IQC:	integral quadratic constraint
KYP, KYPS:	Kalman-Yakubovich-Popov, KYP system
LFC:	local feedback controllers
LFR:	linear fractional representations
LHP (RHP):	left (right) half-complex plane
LMI:	linear matrix inequality
LTI:	linear time-invariant
MIMO:	multi-input multi-output
NFDE:	neutral FDE
ODE:	ordinary differential equations
PDE:	partial differential equations
PEEC:	partial equivalent electric circuit
PSS:	power system stabilizer
RFC:	remote feedback controller
RFDE:	retarded FDE
SDP:	semidefinite programming
SISO:	single-input single-output
SPM:	synchronized phasor measurements
SPR:	strictly-positive real
ssv:	structured singular value
TCP/IP:	transfer control protocol / internet protocol
VLSI:	very large scale integrated
UPO:	unstable periodic orbits.