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Spectral Finite Element Method

**Wave Propagation, Diagnostics and Control in
Anisotropic and Inhomogeneous Structures**

 Springer

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To our parents and wives

Preface

Wave propagation is an exciting field having applications cutting across many disciplines. In the field of structural engineering and smart structures, wave propagation based tools have found increasing applications especially in the area of structural health monitoring and active control of vibrations and noise. In addition, there has been tremendous progress in the area of material science, wherein a new class of structural materials is designed to meet the particular application. In most cases, these materials are not isotropic as in metallic structures. They are either anisotropic (as in the case of laminated composite structures) or inhomogeneous (as in the case of functionally graded materials). Analysis of these structures is many orders more complex than that of isotropic structures. For many scientists/engineers, a clear difference between structural dynamics and wave propagation is not evident. Traditionally, a structural designer will not be interested in the behavior of structures beyond certain frequencies, which are essentially at the lower end of the frequency scale. For such situations, available general purpose finite element code will satisfy the designer's requirement. However, currently, structures are required to be designed to sustain very complex and harsh loading environments. These loadings are essentially multi-modal phenomena and their analysis falls under the domain of wave propagation rather than structural dynamics. Evaluation of the structural integrity of anisotropic and inhomogeneous structures subjected to such loadings is a complex process. The currently available analysis tools are highly inadequate to handle the modeling of these structures. In this book, we present a technique called the "Spectral Finite Element Method", which we believe will address some of the shortcomings of the existing analysis tools.

Although the spectral finite element method has been in existence for a long time under the name of the dynamic stiffness method, its use was limited to simple vibration studies. It is only in recent times that the potential of this method to handle a wide range of applications has been realized. This is evident from the increasing number of publications in the archival literature. However, we believe that its impact has reached only a small subset

of scientists/engineers working in these areas due to the non-availability of a good textbook. The main aim of this book is to reach out to those analysts/engineers working in new and cutting edge areas to not only highlight the power of this method, but also to serve as a good reference book for specialists.

The spectral finite element method is essentially a finite element method in the frequency domain. In essence, the beauty of the method lies in the fact that one can easily convert a finite element code to a spectral element code without much difficulty. In addition, it uses spectral analysis as a basic tool for element formulation. That is, in the process of element formulation, one can deeply understand the physics behind wave propagation in complex media and its interaction with various boundaries. Frequency domain formulation enables easy and straightforward solution of inverse problems. Hence, the spectral element method can be used as a tool to post-process experimental data.

The book mainly addresses the wave behavior in composites and inhomogeneous media in addition to its application to structural health monitoring and active vibration and wave control. The book introduces new methods for the solution of wavenumbers for propagation in composites and inhomogeneous waveguides. For structural health monitoring, waveguide models for different types of damage are developed. The reader is also introduced to various damage detection schemes that blend well with the spectral element method. Towards the end of the book, a chapter on the use of the spectral element method for active control application is presented.

A step by step modular approach is adopted here in writing this book. A number of numerical results are presented to not only emphasize the efficiency and numerical superiority of the method, but also to bring out the physics of the problem. The reader may notice that in most cases only one element is sufficient for solution of certain problems, where thousands of finite elements are required. The material presented in this book can serve as a graduate level textbook on wave propagation in structures. A separate graduate level course on the spectral finite element method can be developed using this book. This book is written assuming that the reader has only an elementary background in the theory of elasticity, strength of materials, linear algebra and methods for solving ordinary and partial differential equations.

We would like to thank many of the graduate students who have contributed directly or indirectly towards the development of the book. We would particularly thank A. Nag, D. Srikanth, A. Garg and A. Singhal for their contributions.

Bangalore, India
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