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Asymptotic Laws and Methods in Stochastics

A Volume in Honour of Miklós Csörgő

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Carleton hosting international
mathematics symposium July
3–6 to honour 50 years of
Prof. Miklós Csörgő's research

Preface

A Fields Institute International Symposium on Asymptotic Methods in Stochastics was organized and held in honour of Miklós Csörgő's work on the occasion of his 80th birthday at Carleton University, Ottawa, Canada, July 3–6, July 2012. The symposium was hosted and sponsored by the School of Mathematics and Statistics, Carleton University, and co-sponsored by the Fields Institute for Research in Mathematical Sciences.

The symposium attracted more than 70 participants from around the world, including many graduate students and postdoctoral fellows. It is with great sadness that we are to write here that in January 2014, one of the participants, **Marc Yor**, passed away. We recall the happy days we were lucky to spend with him here, while he was attending our conference. We are very pleased that in collaboration with Francis Hirsch and Bernard Roynette, he also contributed a paper for publication in this volume. Unfortunately, we cannot any more thank him for his eminent participation in our symposium, where he also gave a talk on peacocks and associated martingales.

The opening address was given by **Don Dawson**, "Path properties of fifty years of research in Probability and Statistics: a tribute to Miklós Csörgő," that was followed by Miklós presenting his 50- year involvement in *Asymptotic Methods in Stochastics* in a historical context.

In this regard we wish to mention that there were two previous conferences, both held at Carleton University, in celebration of Miklós Csörgő's contributions to Probability and Statistics on the respective occasions of his 65th and 70th birthdays. *The first one*, ICAMPS '97 (International Conference on Asymptotic Methods in Probability and Statistics, 8–13 July 1997), was organized by Barbara Szyszkowicz, who also edited the proceedings volume of this conference (cf. [**V1**] in **Publications of Miklós Csörgő**; bold-face letters and/or numbers in square brackets will throughout refer to the latter list of publications). *The second one*, ICAMS '02 (International Conference on Asymptotic Methods in Stochastics, 23–25 May 2002), was organized by Lajos Horváth and Barbara Szyszkowicz, and, just like the preisent symposium, it was also co-sponsored by Tthe Fields Institute. For the proceedings volume of ICAMS'02, we refer to [**V2**], that is, Volume 44

of Fields Institute Communications, as well as to the there indicated Fields Institute website: www.fields.utoronto.ca/publications/supplements/, where the editors of the latter volume, Lajos Horváth and Barbara Szyszkowicz, also have a 69-page résumé of Miklós' work over the past forty or so years at that time, titled "Path Properties of Forty Years of Research in Probability and Statistics: In Conversation with Miklós Csörgő". This article with its 311 references, together with Miklós' list of publications at that time, is also available as no. 400 – 2004 of the Technical Report Series of LRSP. It can also be accessed on the LRSP website: http://www.lrsp.carleton.ca/trs/trs.html.

We much appreciate having been given the opportunity by the Editorial Board of Publications of the Fields Institute to include in this volume Miklós' above-mentioned list of publications (cf. **Table of Contents**). The Editors also have a 45-page resume, titled "**A Review of Miklós Csörgő's Mathematical Biography**", that can be accessed on the Fields Institute website www.fields.utoronto.ca/publications/supplements/. Unfortunately, due to space limitations, we could not include in this collection our expository style review of SELECTED PATH PROPERTIES OF 50+ YEARS OF RESEARCH IN PROBABILITY AND STATISTICS: IN CONVERSATION WITH MIKLÓS CSÖRGŐ.

In the abstract of his talk at the conference, "Almost exact simulations using Characteristic Functions", **Don McLeish** nicely relates asymptotics, numerical methods and simulations as tools of approximation in Probability and Statistics. We quote the first part of his abstract here:

> Asymptotic statistics explores questions like when and how do functions of observed data behave like functions of normal random variables? and much of the work of Miklós Csörgő and his coauthors can be described analogously as when and how do functionals of an observed path behave like those of corresponding Gaussian processes? . For much of the past century, asymptotics provided the main approximation tool in probability and statistics. Although it is now supplemented with other approximation tools such as numerical methods and simulation, asymptotics remains a key to understanding the behaviour of random phenomena.

The following participants presented 30-minute talks at the conference: Raluca Balan, István Berkes, David Brillinger, Alexander Bulinski, Murray Burke, Endre Csáki, Herold Dehling, Dianliang Deng, Richard Dudley, Shui Feng, Antónia Földes, Peter W. Glynn, Edit Gombay, Karl Grill, Lajos Horváth, Gail B. Ivanoff, Jana Jurečková, Reg Kulperger, Deli Li, Zhengyan Lin, Peter March, Yuliya Martsynyuk, Don McLeish, Masoud Nasari, Emmanuel Parzen, Magda Peligrad, Jon N.K. Rao, Bruno Rémillard, Pál Révész, Murray Rosenblatt, Susana Rubin-Bleuer, Thomas Salisbury, Qi-Man Shao, Zhan Shi, Josef G. Steinebach, Qiying Wang, Martin Wendler, Wei-Biao Wu, Marc Yor, and Hao Yu.

We are pleased to publish this collection of twenty articles in the *Fields Institute Communications* series by Springer, and it is our pleasure to dedicate this volume to Miklós Csörgő as a token of respect and appreciation of his work in Probability and Statistics by all the contributors to this volume, and all the participants in our 2012 Fields Institute International Symposium. We are grateful to the contributors for submitting their papers for publication in this volume, as well as to the referees

for their valuable time and enhancing work on it. All papers have been refereed, and accordingly revised if so requested by the editors. We wish to record here our sincere thanks to everyone for their extra time, care and collaboration throughout this elaborate process. The papers in this volume contain up-to-date surveys and original results at the leading edge of research in their topics written by eminent international experts. They are grouped into seven sections whose headings are indicative of their respective main themes that also reflect Miklós' wide-ranging research areas in Probability and Statistics. Except for Section 2, the listing of the articles in each is in the alphabetical order resulting from that of their authors. The reason for making an exemption from this "rule" in Section 2 is that the Csáki et al. paper there also provides a general footing for the results that are proved in Révész's exposition right after.

In **Section 1**, **Miklós Csörgő** and **Zhishui Hu** present, in a historical context, and then establish, a weak convergence theorem for self-normalized partial sums processes of independent identically distributed summands when the latter belong to the domain of attraction of a stable law with index $\alpha \in (0, 2]$. In particular, Theorem 1.1 of this paper identifies the limiting distribution in Theorem 1.1 of Chistyakov and Götze (cf. 2. in References therein) under the same necessary and sufficient conditions in terms of weak convergence in $D[0, 1]$. Initiated by his primary contributions [**97**] (with Lajos Horváth), [**190**] and [**191**] (both with Barbara Szyszkowicz and Qiying Wang), self-normalization and Studentization have become an important global research area of Miklós Csörgő and his collaborators (cf., e.g., [**192**], [**204**], [**205**], [**216**], [**217**], [**220**], [**221**], [**222**], [**223**] and [**224**]). In the introduction of their paper in this section, **Dianliang Deng** and **Zhitao Hu** present an up-to-date survey of results dealing with the precise asymptotics for the deviation probabilities of self-normalized sums and continue with establishing integrated precise asymptotics results for the general deviation probabilities of multidimensionally indexed self-normalized sums. **Magda Peligrad** and **Hailin Sang** deal with asymptotic results for linear processes in general and, in the latter context, review some recent developments, including the central limit theorem (CLT), functional CLT and their self-normalized forms for partial sums. They study these in terms of independent and identically distributed summands (cf. 16. in References therein) and, via self-normalization, for short memory linear processes as well, as, e.g., in 14. in References therein. Self-normalized CLT and self-normalized functional CLT are also covered for long memory linear processes with regularly varying coefficients (cf. 15. in References therein).

In **Section 2**, **Endre Csáki**, **Antónia Földes** and **Pál Révész** survey their joint work with Miklós on anisotropic random walks on the two-dimensional square lattice $\mathbb{Z}^2$ of the plane (cf. [**210**], [**213**], [**215**], [**218**], and [**219**]). Such random walks possibly have unequal symmetric horizontal and vertical step probabilities, so that these step probabilities can only depend on the value of the vertical coordinate. In particular, if such a random walk is situated at the site on the horizontal line $y = j \in \mathbb{Z}$, then, at the next step, it moves with probability p_j to either vertical neighbour and with probability $1/2 - p_j$, to either horizontal neighbour. It is assumed throughout that $0 < p_j \leq 1/2$ and $\min_{j\in\mathbb{Z}} p_j < 1/2$.

The case $p_j = 1/2$ for some j means that the horizontal line $y = j$ is missing, a possible lack of complete connectivity. The initial motivation for studying such two-dimensional random walks on amnisotropic lattice has originated from the so-called transport phenomena of statistical physics (cf. 12., 14., 15. and 16. in References therein), where having $p_j = 1/2, j = \pm 1, \pm 2, \ldots$, but $p_0 = 1/4$, the so-called random walk on the two-dimensional comb, i.e., when all the horizontal lines of the x axis are removed, is also of interest (cf. 1., 2., 5. and 29. in References therein). In his paper, **Pál Révész** continues the investigation of the latter comb-random walk, and also that of a random walk on a half-plane half-comb lattice (cf. [**218**]), and concludes a result for each on the area of the largest square they respectively succeed in covering at time n. **Gail B. Ivanoff** reviews martingale techniques that play a fundamental role in the analysis of point processes on $[0, \infty)$, and revives the question of applying martingale methods to point processes in higher dimensions. In particular, she revisits the question of a compensator being defined for a planar point process in such a way that it exists, it is unique and it characterizes the distribution of the point process. She proceeds to establish a two-dimensional analogue of Jacod's characterization of the law of a point process via a regenerative formula for its compensator and also poses some related open questions.

In **Section 3**, the paper of **Alexander Bulinski** deals with high -dimensional data that can be viewed as a set of values of some factors and a binary response variable. For example, in medical studies the response variable can describe the state of a patient's health that may depend only on some parts of the factors. An important problem is to determine collections of significant factors. In 3. of References of the paper, Bulinski establishes the basis for the application of the multifactor dimensionality reduction (MDR) method in this regard, when one uses an arbitrary penalty function to describe the prediction error of the binary response variable by means of a function of the factors. The goal of thie present paper is to conclude multidimensional CLT's for statistics that justify the optimal choice of a subcollection of the explanatory variables. Statistical variants of these CLT's involving self-normalization are also explored. The paper of **Deli Li**, **Yongcheng Qi** and **Andrew Rosalsky** is devoted to extending recent theorems of Hechner, and Hechner and Heinkel (cf. 6. and 7. in References therein) dealing with sums of independent Banach space valued random variables. The proof of the main result, Theorem 3 in this paper, is based on new versions of the classical Lévy, Ottaviani, and Hoffmann-Jorgensen inequalities (cf. 11., 3. and 8., respectively, in References therein) that were recently obtained by Li and Rosalsky (cf. 13. in References of the paper). In her second paper in this volume, **Magda Peligrad** surveys the almost sure CLT and its functional form for stationary and ergodic processes. Her survey addresses the question of limit theorems, started at a point, for almost all points. These types of results are also known under the name of quenched limit theorems, or almost sure conditional invariance principles. All these results have in common is that they are obtained via a martingale approximation in the almost sure sense. As applications of the surveyed results, several classes of stochastic processes are shown to satisfy quenched CLT and quenched invariance principles, namely, classes of mixing sequences, shift processes, reversible Markov

Chains and Metropolis Hastings algorithms. In his paper, **Qiying Wang** revisits, with some improvements, his recent extended martingale limit theorem (MLT) and, for a certain class of martingales, concludes that the convergence in probability of the conditional variance condition in the classical MLT can be reduced to the less restrictive convergence in distribution condition for the conditional variance (cf. 7. in References therein). The aim of this paper is to show that the latter extended MLT can be used to investigate a specification test for a nonlinear cointegrating regression model with a stationary error process and a nonstationary regressor. This, in turn, leads to a neat proof for the main result in Wang and Phillips of 11. in References.

Anchored by his 1997 book with Lajos Horváth (cf. [**A5**]), change-point analysis has been an important research area of Miklós and his collaborators for almost three decades now (cf. [**93**], [**94**], [**105**], [**106**], [**110**], [**147**], [**148**], [**155**], [**175**], [**198**], [**204**] and [**223**]). The three papers in **Section 4** present recent advances in the field. **Alina Bazarova**, **István Berkes** and **Lajos Horváth** develop two types of tests to detect changes in the location parameters of dependent observations with infinite variances. In particular, autoregressive processes of order one with independent innovations in the domain of attraction of a stable law of index $\alpha \in (0, 2)$ are considered, and, for testing the null hypothesis of the stability of the location parameter versus the at most one-change alternative, they construct a suitably trimmed CUSUM process via removing the d largest observations from the sample. They recall (cf. 8. in References therein) that the thus adjusted CUSUM process converges weakly to a Brownian bridge, if $d = d(n) \to \infty$ fast enough but so that $d(n)/n \to 0$, as $n \to \infty$. However the normalizing sequence depends heavily on unknown parameters. In view of this, two types of test statistics are studied, namely, maximally selected CUSUM statistics whose long run variance is estimated by kernel estimators, and ratio statistics that do not depend on the long run variances whose estimation is thus avoided. **Herold Dehling**, **Roland Fried**, **Isabel Garcia**, and **Martin Wendler** study the detection of change-points in time series. Instead of using the classical CUSUM statistic for detection of jumps in the mean that is known to be sensitive to outliers, a robust test based on the Wilcoxon two-sample test statistic is proposed. The asymptotic distribution of the proposed test can be derived from a functional central limit theorem for a two-sample U-statistics -dependent data that in the case of independent data was studied by Csörgő and Horváth (cf. 5. in References therein, [**106**] in Miklós' list). In their present paper, their result is extended to short-range-dependent data, namely, data that can be represented as functionals of a mixing process. Similar results were obtained for long-range-dependent data by Dehling, Rooch and Taqqu (cf. 10. in References therein). Further to [**106**], we mention [**204**], where the projection variate is assumed to be in the domain of attraction of the normal law, possibly with infinite variance. **Edit Gombay** deals with retrospective change-point detection in a series of observations generated by a binary time series model with link functions other than the logit link function that was considered by Fokianos, Gombay and Hussein in 5. of References therein that appeared in 2014. It is shown that the results in the latter work carry over if, instead of the logit link function, one uses the probit, the log-log, and complementary log-log link functions in the binary

regression model. Some of the technical details omitted in 5. are also detailed in their present paper.

In **Section 5**, **Kilani Ghoudi** and **Bruno Rémillard** investigate the asymptotic behaviour of multivariate serial empirical and copula processes based on residuals of autoregressive-moving-average (ARMA) models. Motivated by Genest et al. 14. as in References therein, multivariate empirical processes based on squared and other functions of residuals are also investigated. Under the additional assumption of symmetry about zero of the innovations, it is shown that the limiting processes are parameter-free. This, in turn, leads to developing distribution-free nonparametric tests for a change-point in the distribution of the innovations, tests of goodness-of-fit for the law of innovations, and tests of independence for m consecutive innovations. Simulations are also carried out to assess the finite-sample properties of the proposed tests and to provide tables of critical values. **Murray Rosenblatt** presents a historical overview of the evolution of a notion of strong mixing as a measure of short-range dependence and a sufficient condition for a CLT. He also discusses a characterization of strong mixing for stationary Gaussian sequences, as well as examples of long-range dependence leading to limit theorems with nonnormal limiting distributions. Results concerning the finite Fourier transform are noted, and a number of open questions are considered. We also note in passing that the articles [**197**], [**200**], [**206**], [**207**], [**214**] and [**227**] in Miklós' list of publications deal with empirical and partial sums processes that are based on short and long memory sequences of random variables.

In **Section 6**, the paper by **Hongwei Dai**, **Donald Dawson** and **Yiqiang Zhao** extends the classical kernel method employed for two-dimensional discrete random walks with reflecting boundaries. The main focus of the paper is to provide a survey on how one can extend the latter kernel method to study asymptotic properties of stationary measures for continuous random walks. The semimartingale reflecting Brownian motion is taken as a concrete example to detail all key steps in the extension in hand that is seen to be completely parallel to the method for discrete random walks. The key components in the analysis for a boundary measure, including analytic continuation, interlace between the two boundary measures, and singularity analysis, allow the authors to completely characterize the tail behaviour of the boundary measure via a Tauberian-like theorem. In their paper, **Peter Glynn** and **Rob Wang** develop central limit theorems and large deviation results for additive functionals of reflecting diffusion processes that incorporate the cumulative amount of boundary reflection that has occurred. In particular, applying stochastic calculus and martingale ideas, partial differential equations are derived from which the central limit and law of large numbers behaviour for additive functionals involving boundary terms can be computed. The corresponding large deviation theory for such additive functionals is then also developed. For papers on additive functionals in Miklós' list of publications, we refer to [**134**], [**152**], [**187**] and [**212**]. Paper [**173**] in the same list contains a self-contained background on stochastic analysis, Itô calculus included. The paper by **Francis Hirsch, Bernard Roynette** and **Marc Yor** studies peacock processes which play an important role in mathematical finance. A deep theorem of Kellerer (cf. 9. in References therein)

asserts the existence of peacock processes as a Markov martingale with given marginals, assumed to increase in the convex order. The paper in hand revisits Kellerer's theorem with a proof, in the light of the papers 5. and 8. in its References by Hirsch-Roynette and G. Lowter, respectively, and presents, without proof, results of 6., 7. and 8. by G. Lowter, which complete and make Kellerer's theorem more precise on some points. Many other references around Kellerer's theorem can be found in 4. of References of the paper.

Mayer Alvo's paper in **Section 7** deals with applying empirical likelihood methods to various problems in two-way layouts involving the use of ranks. Specifically, it is shown that the resulting test statistics are asymptotically equivalent to well-known statistics such as the Friedman test for concordance. It is also shown that empirical likelihood methods can be applied to the two-sample problem, as well as to various block design situations. In her paper **Jana Jurečková** highlights asymptotic behaviour in statistical estimation via describing some of the most distinctive differences between the asymptotic and finite-sample properties of estimators, mainly of robust ones. The latter are, in general, believed to be resistant to heavy-tailed distributions, but they can themselves be heavy-tailed. Indeed, as pointed out by Jurečková, many are not finite-sample admissible for any distribution, though they are asymptoticallly admissible. Hence, and also in view of some other examples she deals with in her paper, she rightly argues that before taking a recourse to asymptotics, we should analyzse the finite-sample behaviour of an estimator, whenever possible.

The Fields Institute announcement of our Ssymposium was also noticed by Dr. László Pordány, Aambassador for Hungary in Canada (2012). Seeing the programme, he wrote to Miklós, conveying his wish to receive the Hungarian participants of the conference in his ambassadorial residence. We, in turn, reciprocated with an invitation to His Excellency to attend, and also participate in, the opening of the symposium, that he gracefully accepted. Following the first -day programme, in the evening, Ambassador László Pordány and Mrs. Mária Csikós welcomed the Hungarian participants at the Aambassador's residence, and His Excellency used the occasion to speak *In Memoriam* **Sándor Csörgő** (Egerfarmos, 16 July 16, 1947 – Szeged, 14 February 14, 2008). We most sincerely thank Dr. László Pordány for the eminent role he played in making the first day of our conference especially memorable.

The occasion of presenting this volume also gives us the opportunity to sincerely thank the Fields Institute for Research in Mathematical Sciences for their financial support of our symposium. We hope very much that this volume, and the national and international success of our conference itself, will have justified their much appreciated trust in us.

Last, but not least, we most sincerely wish to thank Gillian Murray, the coordinator of our manifold LRSP (Laboratory for Research in Statistics and Probability) activities for more than three decades, for her help in preparing this volume, in collaboration with Rafal Kulik and Barbara Szyszkowicz, for publication, while in retirement now.

In conclusion, we also want to express our appreciation to the Editorial Board of the Fields Institute for their approval of the publication of these proceedings in their Communications series; to Carl R. Riehm, the Managing Editor of Publications, for his kind attention to, and sincere interest in, the publication of this volume, and to the Publications Manager, Debbie Iscoe, for her cooperation and expert help in its preparation for Springer. We hope very much that the readers will find this collection of papers, and our introductory comments on them, informative and also of interest in their studies and research work in Stochastics.

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References

1. *Asymptotic Methods in Probability and Statistics: A volume in Honour of Miklós Csörgő* (B. Szyszkowicz, Ed.). Elsevier, Amsterdam 1998.
2. Path Properties of Forty Years in Probability and Statistics: in Conversation with Miklós Csörgő, the Editors: Lajos Horváth and Barbara Szyszkowicz. *Technical Report Series of LRSP, No.* 400, *Carleton University-University of Ottawa* 2004.
3. *Asymptotic Methods in Stochastics: Festschrift for Miklós Csörgő* (L. Horváth, B. Szyszkowicz, Eds.). Fields Institute Communications, Volume 44, AMS 2004.

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