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Hamiltonian Partial Differential Equations and Applications

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Preface

Partial differential equations (PDEs) are a fundamental tool in the modeling of phenomena arising in the physical sciences. PDEs with Hamiltonian structure are a distinguished subset, which not only model systems with conserved quantities (e.g., energy and momentum), but also possess an array of special techniques for their analysis and simulation. They constitute an active area of research where major innovations have been and continue to be made from both the mathematical and computational sides. Not only do these innovations benefit the field itself but also contribute to progress in a vast range of other scientific areas. Applications of Hamiltonian PDEs are numerous in fluid mechanics, plasma physics, and nonlinear optics with such notable examples as the Korteweg–de Vries equation and the nonlinear Schrödinger equation.

In the last few decades, significant progress has been achieved in the mathematical study of these evolutionary PDEs by adopting the "dynamical systems" approach, extending refined analytical techniques of Hamiltonian dynamical systems to the setting of PDEs. This point of view has led to the consideration of the global behavior of orbits for a Hamiltonian PDE in an appropriate phase space, the pursuit of the mathematical technology of normal forms, the study of stable orbits and Kolmogorov–Arnold–Moser (KAM) tori, and a number of results analogous to Nekhoroshev stability and Arnold diffusion. In particular, building on the experience gained from the qualitative study of finite-dimensional dynamical systems, the search for periodic and quasi-periodic solutions has been regarded as a first step towards better understanding the complicated flow evolution of Hamiltonian PDEs. A central tool is transformation theory including Birkhoff normal form transformations. In the broad picture, the goal is to understand some of the important structures of infinite-dimensional phase spaces in which these evolutionary equations are naturally posed, such as periodic orbits, embedded invariant tori, center manifolds, and the different effects of resonances in the non-compact versus compact cases. Techniques from transformation theory for Hamiltonian PDEs with a small parameter have also been successfully used in recent work on water waves, allowing for the systematic derivation of Hamiltonian models in various asymptotic limits.

On 10–12 January 2014, a conference on "Hamiltonian PDEs: Analysis, Computations and Applications" was held at the Fields Institute for Research in Mathematical Sciences in Toronto, bringing together a group of world-class researchers to present and discuss the latest developments in this field. Given the wide range of applications and mathematical tools, a motivating theme of this event was the interaction of specialists in dynamical systems, KAM theory, normal form theory, PDE theory and variational methods, as well as applied and numerical analysts, and experts in water waves. The program consisted of eighteen lectures by distinguished faculty, together with three shorter presentations by junior speakers including two graduate students. The participants came from Canada, Europe, and the USA.

This conference was also an opportunity to honor our friend and colleague Walter Craig, who has made significant contributions to this field, on the occasion of his 60th birthday. Walter obtained his Ph.D. degree from the Courant Institute of Mathematical Sciences (NYU) in 1981. He has held faculty positions at CalTech, Stanford University and Brown University before joining McMaster University as a Professor and Canada Research Chair of Mathematical Analysis and its Applications. He has received a number of prestigious awards including an Alfred P. Sloan Fellowship, an NSF Presidential Young Investigator Award, a Killam Research Fellowship and is a Fellow of the AMS, the AAAS, the Fields Institute and the Royal Society of Canada. He has served on the editorial boards of several journals including the Philosophical Transactions of the Royal Society, the Proceedings of the AMS, and the SIAM Journal on Mathematical Analysis. Walter is a world-renowned mathematical analyst with interests in nonlinear PDEs, Hamiltonian dynamical systems and their physical applications. He has authored more than 100 research articles.

This special volume presents a unique selection of both survey and original research papers by experts who participated in that conference. The various topics discussed in this volume are representative of the wide scope covered by Hamiltonian PDEs, and the results range from mathematical modeling to rigorous analysis and numerical simulation. These topics also reflect Walter Craig's breadth

in research interests and his influence in this field. This book will be of particular interest to graduate students as well as researchers in mathematics, physics, and engineering, who wish to learn more about the powerful and elegant analytical techniques for Hamiltonian PDEs.

The editors would like to thank the Fields Institute for Research in Mathematical Sciences and the Department of Mathematics & Statistics at McMaster University for their generous support. In particular, we are grateful to Alison Conway, Drs. Matheus Grasselli and Hans Boden for their assistance with the organization of the conference, as well as to Debbie Iscoe, Dr. Carl Riehm, and the Springer team for their assistance with the publication of this special volume. We are also thankful to the authors for contributing such excellent articles and to the referees for their invaluable help during the review process. Finally, we dedicate this book to Walter Craig who has been a constant source of inspiration, and whose enthusiasm and friendship have never waned. We would like to extend to him our warmest wishes for many more happy events to come.

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