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Lev Kantorovich

Mathematics for Natural Scientists

Fundamentals and Basics

Springer

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Preface

The idea to write a mathematics textbook for physics undergraduate students presented itself only after more than 10 years of teaching a mathematics course for the second year students at King's College London. I discovered that students starting their studies at university level are lacking elementary knowledge of the fundamentals of mathematics; yet in their first year, this problem is normally left untreated. Instead, the assumption is made that a baseline understanding of numbers, functions, numerical sequences, differentiation and integration was adequately covered in the school curriculum and thus they remain poorly represented at university. Consequently, the material presented to first years is almost exclusively new and leaves the students wondering where it had all come from and struggling to follow the logic behind it. Students, for instance, have no understanding of "a proof". They are simply not familiar with this concept as everything they have learned so far was presented as "given", which is a product of the spoon-feeding teaching methods implemented in so many schools. In fact, the whole approach to teaching mathematics (and physics) at school is largely based on memorising facts. How can one learn, appreciate and apply mathematics in unknown situations if the foundations of what we now call mathematics, formed from lines of thought by some of mankind's best minds over the centuries, remain completely untouched? How can we nurture independent thinkers, thoughtful engineers, physicists, bankers and so on if they can only apply what is known with little ability to create new things and develop new ideas? We are raising a generation of dependents. This can only be avoided if students are taught how to think, and part of this learning must come from understanding the foundations of the most beautiful of all sciences, mathematics, which is based on a magnificent edifice of logic and rigour.

One may hope, naturally, that students can always find a book and read. Unfortunately, very little exists in the offering today for physics students. Indeed, there are some good books currently being used as textbooks for physics and engineering students [1–5], which give a broad account of various mathematical facts and notions with a lot of examples (including those specifically from physics) and problems for students to practice on. However, these books in their presentation lack rigour; most of the important statements (or theorems) are not even mentioned while others are discussed rather briefly and far too “intuitively”. Most worryingly, all these books assume previous knowledge by the student (e.g. of limits, derivatives and integrals) and start immediately from the advanced material leaving essentially an insurmountable vacuum in students’ knowledge of fundamental principles.

I myself studied physics and mathematics between 1974 and 1979 at the Latvian University in Riga (the capital of Latvia, formerly USSR) where a very different approach to teaching students was adopted. We were taught by active research mathematicians (rather than physicists) and at a very different level to the one which is being widely used today at western universities. The whole first year of studies was almost completely devoted to mathematics and general physics; the latter was an introductory course with little mathematics, but covering most of physics from mechanics to optics. Concerning mathematics, we started from the very beginning: from logic, manifolds and numbers, then moving on to functions, limits, derivatives and so on, proving every essential concept as we went along. There were several excellent textbooks available to us at the time, e.g. [6, 7], which I believe are still among the best available today, especially for a student yearning for something thought provoking. Unfortunately, the majority of these books are inaccessible for most western students as they are only available in Russian, or are out of print. In this project, an attempt has been made to build mathematics from numbers up to functions of a single and then of many variables, including linear algebra and theory of functions of complex variables, step-by-step. The idea is that the order of introduction of material would go gradually from basic concepts to main theorems to examples and then problems. Practically everything is accompanied by proofs with sufficient rigour. More intuitive proofs or no proofs at all are given only in a small number of exceptional cases, which are felt to be less important for the general understanding of the concept. At the same time, the material must be written in such a way that it is not intimidating and is easy to read by an average physics student and thus a decision was made not to overload the text with notations mathematicians would find commonplace in their literature. For instance, symbols such as $\in$, $\ni$, $\subset$, $\supset$, $\cup$, etc. of mathematical logic are not used although they would have enabled in many places (e.g. in formulating theorems and during proofs) to shorten the wording considerably. It was felt that this would make the presentation very formal and would repel some physics students who search for something straightforward and palpable behind the formal notation. As a result, the material is presented by a language that is more accessible and hence should be easier to understand but without compromising the clarity and rigour of the subject.

The plan was also to add many examples and problems for each topic. Though adding examples is not that difficult, adding many problems turned out to be a formidable task requiring considerable time. In the end, a compromise was reached to have problems just sufficient for a student reading the book to understand the material and check him/herself. In addition, some of the problems were designed to illustrate the theoretical material, appearing at the corresponding locations throughout the text. This approach permitted having a concise text of reasonable size and a correspondingly sensible weight for the hardcopy book. Nevertheless, there are up to a hundred problems in each chapter, and almost all of them are accompanied by answers. I do not believe that not having hundreds of problems for each section (and therefore several hundreds in a chapter) is a significant deficiency of this book as it serves its own specific purpose of filling the particular gap in the whole concept of teaching mathematics today for physics students. There are many other books available now to students, which contain a large number of problems, e.g. [1–5], and hence students should be able to use them much more easily as an accompaniment to the explanations given here.

As work progressed on this project, it became clear that there was too much material to form a single book and so it was split into two. This one you are holding now is the first book. It presents the basics of mathematics, such as numbers, operations, algebra, some geometry (all in Chap. 1), and moves on to functions and limits (Chap. 2), differentiation (Chaps. 3 and 5), integration (Chaps. 4 and 6), numerical sequences (Chap. 7) and ordinary differential equations (Chap. 8). As we build up our mathematical tools, examples from physics start illustrating the maths being developed whenever possible. I really believe that this approach would enable students to appreciate mathematics more and more as the only language which physics speaks. This is a win-win situation as on the other hand, a more in-depth knowledge of physics may follow by looking more closely at the mathematics side of a known physics problem. Ultimately, this approach may help the students to better apply mathematics in their future research or work.

In the second book, more advanced material will be presented such as: linear algebra, Fourier series, integral transforms (Fourier and Laplace), functions of complex variable, special functions including general theory of orthogonal polynomials, general theory of curvilinear coordinates including their applications in differential calculus, partial differential equations of mathematical physics, and finally calculus of variation. Many more examples from physics will be given there as well. Both volumes taken together should comprise a comprehensive collection of mathematical wisdom hopefully presented in a clear, gradual and convincing manner, with real illustrations from physics. I sincerely hope that these volumes will be found sufficient to nurture future physicists, engineers and applied mathematicians, all of which are desperately lacking in our society, particularly those with a solid background in mathematics.

I am also convinced that the book should serve well as a reach reference material for lecturers. In spite of a relatively small number of pages in this volume, there really is a lot of in-depth material for the lecturers to choose from.

In preparing the book, I have consulted extensively a number of excellent existing books [6–9], most of them written originally in Russian. Some of these are the same books I used to learn from when I was a student myself, and this is probably the main reason for using them. To make the reading easier, I do not cite the specific source I used in the text, so I'd like to acknowledge the above-mentioned books in general here. A diligent student may want to get hold of those books for further reading to continue his/her education. In addition, I would also like to acknowledge the invaluable help I have been having along the way from Wikipedia, and therefore my gratitude goes to all those scholars across the globe who contributed to this fantastic online resource.

I would also like to thank my teachers and lecturers from the Latvian University who taught me mathematics there and whose respect, patience and love for this subject planted the same feeling in my heart. These are, first of all, Era Lepina who taught me the foundations of analysis during my first year at University (I still keep the notes from her excellent lectures and frequently consulted them during the work on this book!), Michael Belov and Teodors Cirulis (theory of functions of complex variables and some other advanced topics such as asymptotic series). I would also like to mention some of my physics lecturers such as Boris Zapol (quantum mechanics), Vladimir Kuzovkov (solid state physics) and Vladimir Ivin (group theory) who made me appreciate mathematics even more as the universal language of physics.

Finally, I would like to apologise to the reader for possible misprints and errors in the book which are inevitable for the text of this size in spite of all the efforts to avoid these. Please send your comments, corrections and any criticism related to this book either to the publisher or directly to myself (lev.kantorovitch@kcl.ac.uk). Happy reading!

London, UK Lev Kantorovich

References

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8. Шипачев ВС (1998) Высшая математика, Москва, Высшая Школа (in Russian). [Shipachev VS (1998) Higher mathematics Vishaja Shkola] I was not able to find an English translation of this book
9. Widder DV (1989) Advanced calculus, 2nd edn. Dover, New York ISBN-13 978-0-486-66103-2

Famous Scientists Mentioned in the Book

Throughout the book various people, both mathematicians and physicists, who are remembered for their outstanding contribution in developing science, will be mentioned. For reader's convenience their names (together with some information borrowed from their Wikipedia pages) are listed here in the order they first appear in the text:

René Descartes (Latinized: **Renatus Cartesius**) (1596–1650) was a French philosopher, mathematician and writer.

Pythagoras of Samos (c. 570 BC – c. 495 BC) was an Ionian Greek philosopher, mathematician and founder of the religious movement called Pythagoreanism.

Tullio Levi-Civita (1873–1941) was an Italian mathematician.

Leopold Kronecker (1823–1891) was a German mathematician who worked in the areas of number theory and algebra.

Gerolamo (or **Girolamo**, or **Geronimo**) **Cardano** (1501–1576) was an Italian mathematician, physician, astrologer and gambler.

Rafael Bombelli (1526–1572) was an Italian mathematician.

William Rowan Hamilton (1805–1865) was an Irish mathematician, physicist and astronomer.

Blaise Pascal (1623–1662) who was a French mathematician, physicist, inventor, writer and Christian philosopher.

Abu Bakr ibn Muhammad ibn al Husayn al-Karaji (or **al-Karkhi**) was a Persian mathematician and engineer.

Baron **Augustin-Louis Cauchy** (1789–1857) was a famous French mathematician who contributed in many areas of mathematics.

Viktor Yakovych Bunyakovsky (1804–1889) was a Ukrainian mathematician.

Karl Hermann Amandus Schwarz (1843–1921) was a famous German mathematician.

Guido Grandi (1671–1742) was an Italian mathematician.

Oliver Heaviside (1850–1925) was a self-taught English electrical engineer, mathematician and physicist.

Lodovico Ferrari (1522–1565) was an Italian mathematician.

Niccolo Fontana Tartaglia (1499/1500–1557) was an Italian mathematician.

Gerolamo Cardano (1501–1576) was an Italian mathematician.

Paolo Ruffini (1765–1822) was an Italian mathematician.

Niels Henrik Abel (1802–1829) was a Norwegian mathematician. He is famously known for proving that the solution of the quintic algebraic equation cannot be represented by radicals.

Abraham de Moivre (1667–1754) was a French mathematician.

Due to Heinrich Eduard Heine (1821–1881) was a German mathematician.

Bernhard Placidus Johann Nepomuk Bolzano (1781–1848) was a Czech mathematician, philosopher and theologist.

Sir **Isaac Newton** PRS MP (1642–1726/7) was an outstanding English scientist who contributed in many areas of physics (especially, mechanics) and mathematics (calculus).

Sir **William Rowan Hamilton** (1805–1865) was an Irish physicist, astronomer and mathematician.

Gottfried Wilhelm von Leibniz (1646–1716) was a German mathematician and philosopher.

Brook Taylor (1685–1731) was an English mathematician who is best known for Taylor's theorem and Taylor's series.

Michel Rolle (1652–1719) was a French mathematician.

Colin Maclaurin (1698–1746) was a Scottish mathematician.

Joseph-Louis Lagrange (1736–1813) was an Italian Enlightenment Era mathematician and astronomer. He made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.

Guillaume François Antoine, Marquis de l'Hôpital (1661–1704) was a French mathematician.

Johannes Diderik van der Waals (1837–1923) was a Dutch theoretical physicist and thermodynamicist.

Lev Davidovich Landau (1908–1968) was a prominent Soviet physicist who made fundamental contributions to many areas of theoretical physics, 1962 Nobel laureate. He is a co-author of a famous ten volume course of theoretical physics.

Vitaly Lazarevich Ginzburg (1916–2009) was a Soviet and Russian theoretical physicist, astrophysicist, 2003 Nobel laureate.

Georg Friedrich Bernhard Riemann (1826–1866) was a German mathematician.

Thomas Joannes Stieltjes (1856–1894) was a Dutch mathematician.

Henri Léon Lebesgue (1875–1941) was a French mathematician.

Jean-Gaston Darboux (1842–1917) was a French mathematician.

Jean Baptiste Joseph Fourier (1768–1830) was a French mathematician and physicist, best known for Fourier series and integral.

Leonhard Euler (1707–1783) was a famous Swiss mathematician and physicist. He made important discoveries in various fields of mathematics and introduced much of the modern mathematical terminology and notation. He is also renowned for his work in mechanics, fluid dynamics, optics, astronomy and music theory.

Paul Langevin (1872–1946) was a prominent French physicist.

Ludwig Eduard Boltzmann (1844–1906) was an Austrian physicist and philosopher famously known for his fundamental work in statistical mechanics and kinetics.

Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto (1776–1856) was an Italian scientist.

Jean-Baptiste Biot (1774–1862) was a French physicist, astronomer and mathematician.

Félix Savart (1791–1841) was a French physicist.

Pierre-Simon, marquis de Laplace (1749–1827) who was a French mathematician and astronomer.

Hermann Ludwig Ferdinand von Helmholtz (1821–1894) was a German physician and physicist.

Josiah Willard Gibbs (1839–1903) was an American physicist, chemist and mathematician.

James Clerk Maxwell (1831–1879) was a Scottish mathematical physicist best known for his unification of electricity and magnetism into a single theory of electromagnetism.

Karl Theodor Wilhelm Weierstrass (1815–1897) was an outstanding German mathematician contributed immensely into modern analysis.

Carl Gustav Jacob Jacobi (1804–1851) who was a German mathematician.

Andrey (Andrei) Andreyevich Markov (1856–1922) was a Russian mathematician.

Albert Einstein (1879–1955) was a German-born theoretical physicist famously known for his relativity and gravity theories.

Marian Smoluchowski (1872–1917) was an Austro-Hungarian Empire scientist of a Polish origin.

Andrey Nikolaevich Kolmogorov (1903–1987) was a Soviet mathematician, one of the founders of modern probability theory.

Sydney Chapman (1888–1970) was a British mathematician and geophysicist.

George Green (1793–1841) was a British mathematical physicist.

August Ferdinand Möbius (1790–1868) was a German mathematician and theoretical astronomer.

Johann Benedict Listing (1808–1882) was a German mathematician.

George Gabriel Stokes (1819–1903) was an Irish and British mathematician, physicist, politician and theologian.

Mikhail Vasilyevich Ostrogradsky (1801–1862) was a Russian - Ukrainian mathematician, mechanician and physicist.

Johann Carl Friedrich Gauss (1777–1855) was a German mathematician and physicist.

Archimedes of Syracuse (c. 287 BC–c. 212 BC) was a Greek mathematician, physicist, engineer, inventor and astronomer.

Siméon Denis Poisson (1781–1840) was a French mathematician, geometer and physicist.

Hendrik Antoon Lorentz (1853–928) was a Dutch physicist.

André-Marie Ampère (1775–1836) was a French physicist and mathematician.

Michael Faraday (1791–1867) was a famous English physicist.

Adolf Eugen Fick (1829–1901) was a German-born physician and physiologist.

Jean-Baptiste le Rond d'Alembert (1717–1783) was a French mathematician, physicist, philosopher.

Lorenzo Mascheroni (1750–1800) was an Italian mathematician.

Douglas Rayner Hartree (1897–1958) was an English mathematician and physicist.

Max Karl Ernst Ludwig Planck (1858–1947) was a German theoretical physicist.

Satyendra Nath Bose (1894–1974) was an Indian Bengali physicist specialising in mathematical physics.

Adrien-Marie Legendre (1752–1833) was a French mathematician.

Charles Hermite (1822–1901) was a French mathematician.

Edmond Nicolas Laguerre (1834–1886) was a French mathematician.

Pafnuty Lvovich Chebyshev (1821–1894) was a Russian mathematician.

Augustin-Jean Fresnel (1788–1827) was a French engineer known for his significant contribution to the theory of wave optics.

Józef Maria Hoene-Wroński (1776–1853) was a Polish Messianist philosopher who worked in many fields of knowledge, including mathematics and physics.

Erwin Rudolf Josef Alexander Schrödinger (1887–1961) was an Austrian physicist, one of the founders of quantum theory, he is famously known for Schrödinger (wave) equation of quantum mechanics. He also contributed in other fields fo physics such as statistical mechanics and thermodynamics, physics of dielectrics, colour theory, electrodynamics, general relativity and cosmology.

Ferdinand Georg Frobenius (1849–1917) was a German mathematician.

Friedrich Wilhelm Bessel (1784–1846) was a German mathematician and astronomer.

Gustav Robert Kirchhoff (1824–1887) was a German physicist who contributed to the fundamental understanding of electrical circuits, spectroscopy and the emission of black-body radiation by heated objects.

Konstantin Eduardovich Tsiolkovsky (1857–1935) was a Russian and Soviet rocket scientist and pioneer of the astronautic theory.

Svante August Arrhenius (1859–1927) was a Swedish physicist, one of the founders of physical chemistry.

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