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Rigidity and Symmetry

FIELDS The Fields Institute for Research in the Mathematical Sciences

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Preface

The thematic program on Discrete Geometry and Applications took place at the Fields Institute for Research in Mathematical Sciences in Toronto between July 1 and December 31, 2011. The papers included in this book are based on some research conducted during the semester and on some of the lectures there, in particular those related to the part of the program under the heading "Rigidity and Symmetry".

This includes the study of the theory of rigidity as applied to discrete objects such as bar and joint frameworks, tensegrities, body and bar frameworks especially including such symmetric objects, periodic frameworks, and the combinatorics when the objects are symmetric. When the configuration of points that define the object is generic, the rigidity properties reduce to combinatorial properties usually of some underlying graph. When the object is symmetric, it automatically becomes non-generic, but nevertheless it is possible to consider the case when the configuration is generic modulo the symmetry group. This leads to a lot of interesting and intricate theory. It is useful to keep in mind that there are two approaches to a symmetric rigid object. Incidental rigidity is when the object is rigid and symmetric, but it is not constrained to stay rigid under a flex. Forced rigidity is when the object is rigid and symmetric, and the symmetry is part of constraints. Both situations occur here.

Another part concerns symmetry as applied to abstract as well as geometric objects. Central to this theme are polytopes, the generalizations of polygons and polyhedra to higher rank (the abstract analogue of dimension). Several articles are devoted to regular maps on surfaces, which are just polyhedra in a general sense. These usually permit operations – replacing faces by different edge-circuits – that change their combinatorial type, an important idea relating different maps. Such operations can be applied in higher rank as well. Regular and chiral polytopes (the latter roughly speaking half-regular) often correspond to interesting groups, particularly simple ones; such connexions are explored in several papers. Variants of regularity, further weakening the condition, also lead to interesting questions. Closely related to polytopes are graphs and complexes; these are the subject of

other articles. More metrical in scope are papers on volume in non-euclidean spaces, symmetric configurations in the plane, and a concept of rigidity of polytopes that provides a bridge to the previous part.

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