

International Series in Operations Research & Management Science

Volume 207

Series Editor:

Frederick S. Hillier
Stanford University, CA, USA

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Optimal Stochastic Scheduling

 Springer

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ISSN 0884-8289

ISSN 2214-7934 (electronic)

ISBN 978-1-4899-7404-4

ISBN 978-1-4899-7405-1 (eBook)

DOI 10.1007/978-1-4899-7405-1

Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2014930759

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Printed on acid-free paper

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Preface

Machine scheduling concerns about how to optimally allocate the limited resources (machines available) to process jobs over time. It is a decision-making process that plays a crucial role in many environments, including manufacturing, logistics, healthcare, communications, and computing systems. In some industries, such as transportation, scheduling is the mission-critical decision that directly determines the effectiveness and even the survival of a business. There have been examples indicating how a good scheduling solution can enable an organization to significantly enhance its efficiency or a system (e.g., an airline) to quickly recover from a major disruption.

Scheduling is a discipline that has been extensively studied for several decades, with various models established and results derived. However, while there is a large literature on scheduling problems, the majority of the research has been devoted to deterministic scheduling in which all attributes of the problem, such as the amount of time required to process a job and the deadline to complete it, are assumed to be exactly known in advance without any uncertainty. Clearly such an assumption is hardly justifiable in practical situations where more often such parameters are not known in advance and can only be estimated with a varying level of uncertainty. In addition, the majority of scheduling problems studied in the literature assume that the machine to be used to process the jobs is continuously available until all jobs are completed. In reality, however, it is a common phenomenon that a machine may break down randomly from time to time.

As Albert Einstein said: “As far as the laws of mathematics refer to reality, they are not certain, as far as they are certain, they do not refer to reality.” Research interests have been increasingly devoted to *stochastic scheduling* in recent years, which incorporates the approaches of probability and stochastic processes into scheduling problems to account for uncertainties from different sources. Many interesting and important results on stochastic scheduling problems have been developed, with the aid of probability theory. The main purpose of this book is to provide a comprehensive and unified coverage of studies in this area. Our objective is two-fold: (i) to summarize the elementary models and results in stochastic scheduling, so as to offer an entry-level reading material for students to learn and understand the fundamentals of this area; and (ii) to include in details the latest developments and research

topics on stochastic scheduling, so as to provide a useful reference for researchers and practitioners who are performing research and development work in this area.

Accordingly, the materials of this book are organized into two clusters: Chaps. 1–4 cover more fundamental models and results, whereas Chaps. 5–10 elaborate on more advanced topics. Specifically, In Chap. 1, we first provide the relevant basic theory of probability, and then introduce the basic concepts and notation of stochastic scheduling. In Chaps. 2 and 3, we review those well-established models and scheduling policies, under regular and irregular performance measures, respectively. Chapter 4 describes models with stochastic machine breakdowns. Chapters 5 and 6 introduce, respectively, the optimal stopping problems and the multi-armed bandit processes, which are necessary for studies of more advanced subjects. Chapter 7 is focused on dynamic policies. Chapter 8 describes stochastic scheduling with incomplete information, where the probability distributions of random variables contain also unknown parameters, which can however be estimated progressively according to updated information. Chapter 9 is devoted to the situation where the processing time of a job depends on the time when it is started. Lastly, in Chap. 10 we describe several recent models beyond those in Chaps. 1–9.

This book is intended for researchers, practitioners, and graduate students and senior-year undergraduates as a unified reference and textbook on optimal stochastic scheduling. While the various topics are presented within the general framework of stochastic scheduling, we will try to make each chapter relatively self-contained so that they can be read separately. Also, for each model presented, apart from the formulation of the model and the descriptions of the relevant properties and scheduling policies, we will try to provide as much as possible discussion on the open questions and the likely directions for further research.

The publication of this book would not be possible without the help and generous support of many people and organizations. First, we would like to express our sincere gratitude to Prof. Fred Hillier, the Editor of Springer's book series in Operations Research and Management Science, for his encouragement and in particular his patience to wait for the completion of our manuscript. We are indebted to the publishers and staff members of Springer, for their support for us to complete this book project. Many of our colleagues and students have kindly provided us with invaluable comments and suggestions in various occasions such as seminars and conferences. Part of our researches that comprise several chapters in this book have been financially supported by the Research Grants Council of Hong Kong under General Research Fund Nos. 410509 and 410211, Natural Science Foundation of China (NSFC) Grant Nos. 71071056 and 71371074 and Australian Research Council Discovery Project Grant No. DP1094153.

Last but not least, we must express our most sincere gratitude to our families, for their continued and selfless support over the many days and nights during our writing of this book.

Hong Kong
Shanghai
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