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Progress in Physics
Vol. 3

Edited by
A. Jaffe and
D. Ruelle

Springer Science+Business Media, LLC

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Mathematics and Physics

Translated by
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1981 Springer Science+Business Media, LLC

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Library of Congress Cataloging in Publication Data
Manin, IU. I.
Mathematics and Physics.
(Progress in Physics ; 3)
Translation of: Matematika i fizika.

1. Mathematics--1961- . 2. Physics. I. Title.
II. Series: Progress in Physics (Boston, Mass.); 3.
QA37.2.M3413 530.1'5 81-1319
AACR2

CIP — Kurztitelaufnahme der Deutschen Bibliothek
Manin, Jurij I.:
Mathematics and Physics / Yu. I. Manin.
Translated by Ann and Neal Koblitz
Boston: Basel: Stuttgart: Birkhäuser, 1981
(Progress in Physics ; 3)
ISBN 978-1-4899-6784-8
NE:GT

Originally published in Novoye v Zhizny, Naukye, Tekhnike, Seriva Matematika, Kibernetika, Moscow: Znaniye, No.12, 1979.

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© Springer Science+Business Media New York 1981
Originally published by Birkhäuser Boston in 1981
Softcover reprint of the hardcover 1st edition 1981

ISBN 978-1-4899-6784-8 ISBN 978-1-4899-6782-4 (eBook)
DOI 10.1007/978-1-4899-6782-4

CONTENTS

The connection between mathematics and physics... Is it only that physicists talk in the language of mathematics? It is more. This book describes how mathematics associates to some important physical abstractions (models) its own mental constructions, which are far removed from the direct impressions of experience and physical experiment.

FOREWORD

There is a story about how a certain well-known mathematician would begin his sophomore course in logic. "Logic is the science of laws of thought," he would declaim. "Now I must tell you what science is, what law is, and what thought is. But I will not explain what 'of' means."

On undertaking the task of writing the book "Mathematics and Physics", the author realized that its size would hardly be sufficient to attempt to explain what the "and" means in the title. These two sciences, which were once a single branch on the tree of knowledge, have by our time become rather separate. One of the reasons for this is that in this century they have both engaged in introspection and internal development, i.e., they have constructed their own models with their own tools. Physicists were disturbed by the interrelation between thought and reality, while mathematicians were disturbed by the interrelation between thought and formulas. Both of these relations turned out to be more complicated than had previously been thought, and the models, self-portraits, and self-images of the two disciplines have turned out to be very dissimilar. As a result, from their earliest student days mathematicians and physicists are taught to think differently. It would be wonderful to master

both types of professional thinking just as we master the use of both a left and a right hand. But this book is like a melody without accompaniment.

The author, by training a mathematician, once delivered four lectures to students under the title "How a mathematician should study physics". In the lectures he said that modern theoretical physics is a luxuriant, totally Rabelaisian, vigorous world of ideas, and a mathematician can find in it everything to satiate himself except the order to which he is accustomed. Therefore a good method for attuning oneself to the active study of physics is to pretend that you are attempting to induce this very order in it.

In the book, which evolved from these lectures and later reflections, I have tried to select several important abstractions of the two sciences and make them confront one another. At the very highest level such abstractions lose terminological precision and are capable of becoming cultural symbols of the time: we recall the fate of the words "evolution", "relativity", or "the subconscious". Here we go down to a lower level and discuss words which, although not yet symbols, have passed beyond being merely terms: "set", "symmetry", "space-time". (Cf. M.M. Bakhtin's attempt to introduce this last concept into literary criticism through the deliberately foreign term "chronotope".) Some of these words occur in the titles of chapters. Each reader must have in his or her mind preliminary images of these concepts, images which have a physical origin in the broad sense of the word.

The author wishes to show how mathematics associates new mental images with such physical abstractions; these images are almost tangible to the trained mind but are far removed from those that are given directly by life and physical experience. For example, a mathematician represents the motion of the planets of the Solar System by a flow line of an incompressible fluid in a 54-dimensional phase space, whose volume is given by the Liouville measure.

The reader may need an effort of will to perceive mathematics as a tutor of our spatial imagination. More frequently one associates mathematics with rigorous logic and computational formalism. But this is only discipline, a ruler with which we are being taught not to die.†

The computational formalism of mathematics is a thought process that is externalized to such a degree that for a time it becomes alien and is turned into a technological process. A mathematical concept is formed when this thought process, temporarily removed from its human vessel, is transplanted back into a human mold. To think... means to calculate with critical awareness.

The "mad idea" which will lie at the basis of a future fundamental physical theory will come from a realization that physical meaning has some mathematical form not previously associated with reality. From this point of view

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†An allusion to Boris Pasternak's poem "To Briusov", in which the poet says: "There was a time when you yourself each morning/ Taught us with the strict beat of your ruler how not to die." (Translator's note)

the problem of the "mad idea" is a problem of choosing, not of generating, the right idea. One should not understand this too literally. In the 1960's it was said (in a certain connection) that the most important discovery of recent years in physics was the complex numbers. The author has something like that in mind.

I do not wish to apologize for the subjectivity of the opinions and choice of material. Physics and mathematics have been written about by Galileo, Maxwell, Einstein, Poincaré, Feynman, and Wigner, among others; only the hope of saying something subjective of one's own can justify a new attempt.