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Stan Raatz

# Graph-Based Proof Procedures for Horn Clauses

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Stan Raatz  
Department of Computer Science  
Rutgers University  
New Brunswick, NJ 08903  
USA

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## PREFACE

The origins of this monograph lie in my Ph.D. dissertation of 1987 at the University of Pennsylvania, which was concerned with proof procedures for the Horn clause subset of logic. The rise of logic programming has made this an important area of study. All Prologs are based on a variant of resolution, and inherit various properties related to this proof method. This monograph studies the paradigm of logic programming in the context of graph-based proof procedures which are unrelated to resolution.

The monograph is not a general introduction to logic programming, although it is self-contained with respect to the mathematics used. It should appeal to the computer scientist or mathematician interested in the general area we now call computational logic. A large part of the monograph is devoted to detailed proofs that the methods we present are sound and complete, which in the context of the logic programming, means that the operational and denotational semantics agree.

The monograph is organized as follows. After a chapter on mathematical preliminaries, in chapter 3 we present examples of logic programs and define a relevant semantics. The usual semantics of Prolog is not admitted by the methods studied in this work. In chapter 4 we present the mechanics of the first method, Hornlog, which admits one-sorted Horn clause programs consisting of any arbitrary Horn clauses, including clauses of the form  $\leftarrow B_1, \dots, B_n$ , and queries of the form  $Q = \exists x_1 \dots \exists x_n (\neg H_1 \vee \dots \vee \neg H_m)$  where  $\{H_1, \dots, H_m\}$  are again one-sorted Horn clauses. In chapter 5, we give constructive proofs of the soundness and completeness of the answer substitution, and show the relationship between the operational semantics of the method and the model-theoretic semantics of the underlying language.

In chapters 6 and 7, we define an equational extension of Hornlog, the  $HE^\dagger$ -refutation method, which applies to many-sorted first-order equational Horn clause programs consisting of clauses of the form  $s \doteq t$ ,  $A \leftarrow B_1, \dots, B_n$ , or  $\leftarrow B_1, \dots, B_n$  where  $s$  and  $t$  are first-order terms,  $A$  is a non-equational atomic formula, and  $B_1, \dots, B_n$  are either equational or non-equational atomic formulae. This class of programs subsumes the paradigms of functional, logic, and equational programming. In chapter 8, we show the soundness and completeness of this method. Finally, in the appendix chapter 9 we consider the design of an implementation on a abstract parallel machine.

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