Progress in Mathematics

Vol. 1: H. Gross, **Quadratic Forms in Infinite-Dimensional Vector Spaces.** XXII, 419 pages. 1979

In preparation:

C. Okonek, M. Schneider, H. Spindler, **Vector Bundles on Complex Projective Spaces**

This expository treatment of the subject is based on a survey which M. Schneider gave at the Seminaire Bourbaki in November 1978 and on a subsequent course held at the University of Goettingen. It takes into account recent developments and can serve as an introduction to the topical question of classification of holomorphic vector bundles on complex projective spaces. This has become of interest recently to theoretical physicists because of the relationship with instantons.

K. Diederich, **Real Hypersurfaces in C^n**

In this volume, the author gives an expository presentation of the theory of local biholomorphic invariants for real hypersurfaces in C^n.

Progress in Mathematics 1

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To Esther

Preface

For about a decade I have made an effort to study quadratic forms in infinite dimensional vector spaces over arbitrary division rings. Here we present in a systematic fashion half of the results found during this period, to wit, the results on denumerably infinite spaces ("$\aleph_0$- forms"). Certain among the results included here had of course been published at the time when they were found, others appear for the first time (the case, for example, in Chapters IX , X , XII where I include results contained in the Ph.D.theses by my students W. Allenspach, L. Brand, U. Schneider, M. Studer).

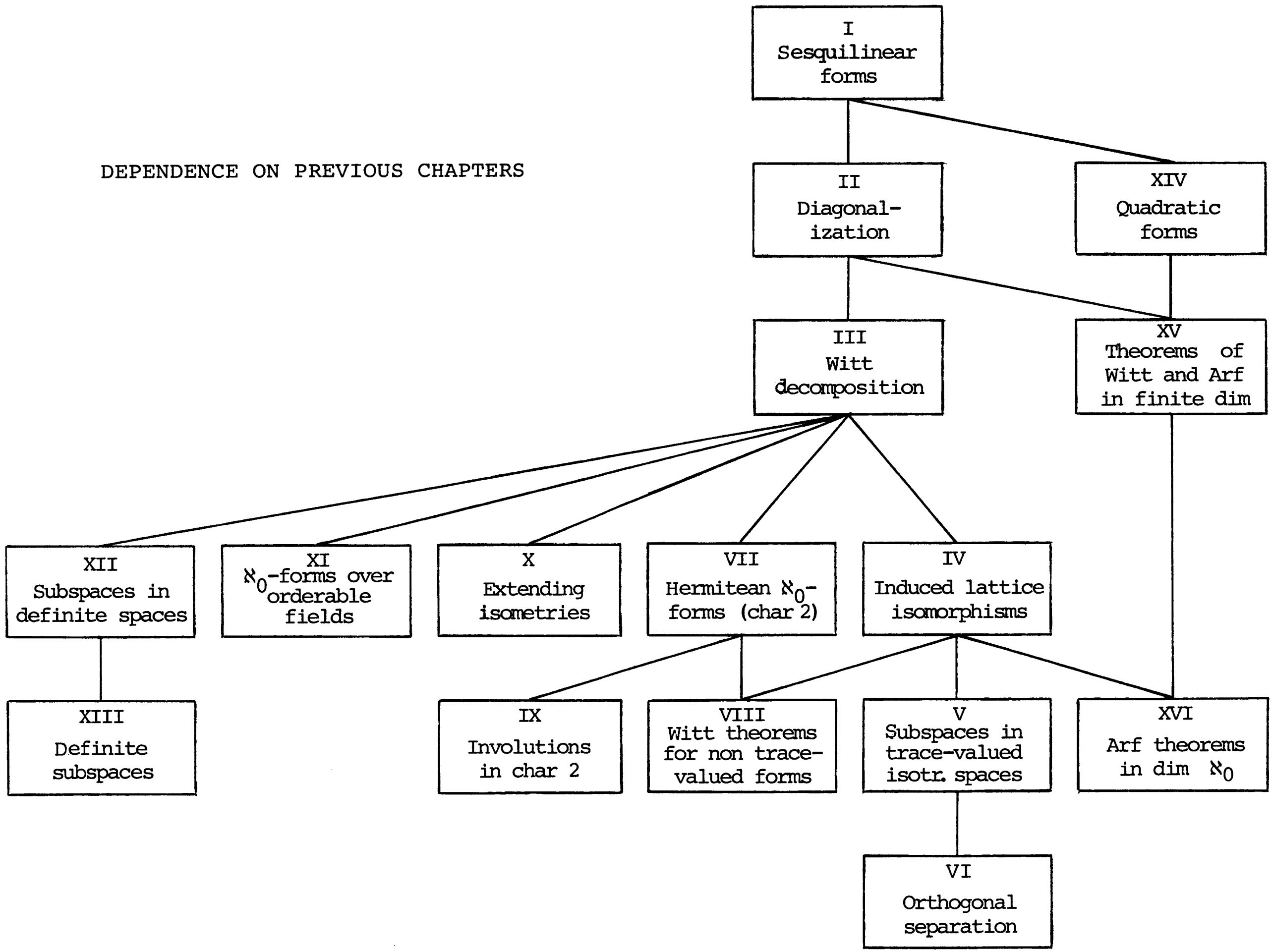
If one wants to give an introduction to the geometric algebra of infinite dimensional quadratic spaces, a discussion of $\aleph_0$- dimensional spaces ideally serves the purpose. First, these spaces show a large number of phenomena typical of infinite dimensional spaces. Second, most proofs can be done by recursion which resembles the familiar procedure by induction in the finite dimensional situation. Third, the student acquires a good feeling for the linear algebra in infinite dimensions because it is impossible to camouflage problems by topological expedients (in dimension $\aleph_0$ it is easy to see, in a given case, whether topological language is appropriate or not).

Two more remarks are in order. Since classical Hilbert spaces have either finite or uncountable dimensions there will be no overlapping with Hilbert space theory here. And, finally, we wish to point out that we have made no steps to generalize away from vector spaces even in cases where such a possibility was in view.

The manuscripts for the book have been critically read and reread by Dr. Werner Bäni. He has eliminated a large number of errors. Yet of greatest importance to me has been his acute mathematical judgement on disputable matters in the texts. I express my warmest thanks to him.

Zurich, March 1979 Herbert Gross

DEPENDENCE ON PREVIOUS CHAPTERS



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