

Practical Python AI Projects

**Mathematical Models of
Optimization Problems with
Google OR-Tools**

Serge Kruk

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Practical Python AI Projects: Mathematical Models of Optimization Problems with Google OR-Tools

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About the Author



Serge Kruk, PhD is a professor at the Department of Mathematics and Statistics at Oakland University and worked for Bell-Northern Research. His current research interests still bear the stamp of practicality enforced by years in industry: algorithms for semidefinite optimization, scheduling, feasibility, and the related numerical linear algebra and analysis. After a few wandering

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