

# Operations Research in Transportation Systems

# Applied Optimization

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Volume 20

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# Operations Research in Transportation Systems

Ideas and Schemes of Optimization Methods  
for Strategic Planning and Operations Management

by

Alexander S. Belenky



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# Preface

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The scientific monograph of a survey kind presented to the reader's attention deals with fundamental ideas and basic schemes of optimization methods that can be effectively used for solving strategic planning and operations management problems related, in particular, to transportation. This monograph is an English translation of a considerable part of the author's book with a similar title that was published in Russian in 1992.

The material of the monograph embraces methods of linear and nonlinear programming; nonsmooth and nonconvex optimization; integer programming, solving problems on graphs, and solving problems with mixed variables; routing, scheduling, solving network flow problems, and solving the transportation problem; stochastic programming, multicriteria optimization, game theory, and optimization on fuzzy sets and under fuzzy goals; optimal control of systems described by ordinary differential equations, partial differential equations, generalized differential equations (differential inclusions), and functional equations with a variable that can assume only discrete values; and some other methods that are based on or adjoin to the listed ones.

The material discussed in the monograph is oriented to problems of a transportation nature and is addressed, first and foremost, to experts working in the field of operations management or strategic planning for transportation. However, its major part devoted to mathematical methods is invariant with respect to a particular application field. Therefore, the monograph may be used by students and postgraduates studying operations management, strategic planning (and related technical and economic sciences), as well as by those studying applied mathematics at universities, technical and economic institutes, and colleges.

A separate chapter contains basic mathematical information (starting from the notions of set theory) necessary for understanding the monograph's material. It makes the presentation of the material closed and formally requires from the reader a certain "culture of thinking" rather than a special mathematical background. At the same time, the author hopes that any reader interested in the acquisition of knowledge can develop such a "culture of thinking" as a

result of work with the monograph, although it will take intellectual effort and time on his part.

A few words about those who helped in giving birth to this monograph. Constant attention and support rendered by B. Polyak stimulated, to a considerable degree, the author's research in optimization and its applications. O. Aven attracted the author's attention to control problems arising in transportation. Fruitful research in scheduling theory applications for transportation systems was started together with E. Levner. Collaboration with these scientists helped the author comprehend a great deal of material in optimization and transportation systems analysis and inspired the idea of writing this monograph. This idea was supported by A. Odoni and A. Kornhauser, and productive discussions with them convinced the author that such a monograph could be helpful for specialists in operations management and strategic planning that apply mathematical methods in the field of transportation. Presentation of the material was substantially improved thanks to the advice and recommendations of N. Tretyakov. The high quality camera-ready copy of the monograph was produced through the diligence of the computer network laboratory's staff (headed by M. Doubson) at the Central Economics and Mathematics Institute of the USSR Academy of Sciences. The author expresses his deep appreciation to all the above-mentioned people.

A few technical remarks on the material's presentation. Wherever possible, the author adhered to notations of the original publications under review. It resulted in employing different (although, traditionally used) notations for the same mathematical subjects in some parts of the monograph. Formulae are numbered independently by chapter. Sources are also numbered independently by chapter and by paragraph within Chapter 3. Those paragraphs may, in fact, be considered as small separate chapters.

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*January, 1992*

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