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DISTORTION ANALYSIS OF ANALOG INTEGRATED CIRCUITS

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SPRINGER SCIENCE+BUSINESS MEDIA, LLC
Foreword

The analysis and prediction of nonlinear behavior in electronic circuits has long been a topic of concern for analog circuit designers. The recent explosion of interest in portable electronics such as cellular telephones, cordless telephones and other applications has served to reinforce the importance of these issues. The need now often arises to predict and optimize the distortion performance of diverse electronic circuit configurations operating in the gigahertz frequency range, where nonlinear reactive effects often dominate. However, there have historically been few sources available from which design engineers could obtain information on analysis techniques suitable for tackling these important problems.

I am sure that the analog circuit design community will thus welcome this work by Dr. Wambacq and Professor Sansen as a major contribution to the analog circuit design literature in the area of distortion analysis of electronic circuits. I am personally looking forward to having a copy readily available for reference when designing integrated circuits for communication systems.

Robert G. Meyer
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1998
Preface

In the world of electronics nowadays very advanced systems can be integrated on one chip. This is mainly possible by the ability to build complex functions with digital VLSI. However, not every functionality can be achieved using digital electronics. For example, some applications might require signal processing at a high frequency that is too high to process with digital circuitry. In that case, the signals must be processed with analog circuitry which can be integrated completely or partially.

Other applications require an interface between the digital electronics and the outside world, which behaves in an analog way. As a result, such interface functions are implemented with analog electronics, which again can be integrated.

The above considerations indicate that analog integrated circuits are required, not only as integrated circuits on their own, but also as part of large mixed-signal integrated circuits. In such mixed-signal systems more and more functions are implemented in a digital way. The specifications of the circuitry that remains to be designed in the analog domain are often very tough. As a result, circuit designers not only need to concentrate on the first-order characteristics of analog circuits, which can already be very complicated and which are most often attributed to the behavior of the linearized circuit. In addition, characteristics such as nonlinear behavior may become very critical.

In addition to mixed-signal applications that demand tough specifications for the analog blocks, there are some applications where the suppression of nonlinear behavior is of utmost importance. Examples are audio applications and telecommunication applications. In the latter applications, nonlinear behavior of the circuits induces intermodulation products, which together with the noise, increase the amount of “unwanted signals”, thereby lowering the performance.

In the last few years, an increased interest is seen in the integration of analog high-frequency front-ends for telecommunication applications, both in CMOS and in BiCMOS. These technologies are quickly scaling to smaller dimensions, such that they can be used at ever increasing frequencies. In this way, these silicon technologies form a cheap alternative for GaAs. CMOS technologies are cheaper than BiCMOS technologies, but the bipolar (npn) transistors of the BiCMOS technologies are in general superior at high frequencies than the MOS transistors. Integrated silicon RF front ends are found in literature for wireless communications [Long 95], used for example in the GSM standard [Seven 91, Stet 95], the DECT standard [Daw 97], and the Japanese standard for the personal handy-phone system (PHS) [Sato 96], further in GPS (Global Positioning System) receivers [Herm 91], wireless local area networks (WLAN) [Madi 96, Har 96], in applications in the ISM bands (Industrial-Scientific-Medical) around 900MHz [Hull 96],
2.4GHz [Mey 97] or 5.7GHz [Voi 97], in the North American Digital Cellular (NADC) handset [Kara 96], and so on.

In analog RF front-ends for communication circuits, specifications that are related to nonlinear circuit behavior are very important. For example, if a receiver front-end is not very linear, then large incoming signals from the antenna will induce much extra distortion. Such large incoming signal may be a wanted signal but it can also be a strong unwanted signal at an adjacent frequency. The increased efforts of the analog design community in the silicon integration of analog high-frequency RF front-ends has been one of the motivations to write this book.

Not only the nonlinear behavior of the RF part of a communication circuit is important to control. In many communication circuits analog integrated filters are used, for example to perform the anti-aliasing filtering function right before an analog-digital converter in a receiver. The nonlinear distortion of these filters can degrade the overall performance of a transceiver. Very often $g_m C$ filters are used in transceivers [Gopi 90, Chang 97], since they can achieve high speeds. This high speed is achieved at the expense of a reduced nonlinearity. In essence, a $g_m C$ cell is an open-loop transconductor. We shall see in this book that the conversion of a voltage into a current, which is realized by a transconductor, is difficult to realize with a high linearity without an overall feedback with a large loop gain. Hence, nonlinear distortion is an important aspect in the design of an active analog integrated filter.

In many applications of analog circuits one is only interested in the circuits' steady-state behavior in response to a sinusoidal excitation or a combination of such excitations. Indeed, many circuit aspects are easier to characterize in the steady state. This is partially due to the fact that an extremely large class of analog circuits can be approximated very well by a linear system. Since sinusoidal functions are eigenfunctions of linear systems, the latter ones can be easily characterized in terms of responses to sinusoidal excitations. Examples of quantities that characterize a circuit in steady state are transfer characteristics like gain or impedances. These characteristics are also best measured when a circuit is in steady state. Gain and impedances are mainly due to the behavior of the linearized circuit. However, most analog integrated circuits behave weakly nonlinearly. This means that, when a sinusoidal signal or a combination of sinusoids is applied to a circuit, the output spectrum does not only contain signals with the same frequency of the input signals, as one would expect of a linear system: in addition, the output spectrum contains small components — usually unwanted — at frequencies other than the input signal frequencies. If one sinusoidal signal is applied at the input, then these unwanted signals occur at multiples of one of the input frequencies. In this case they are termed as harmonics. In the case of an excitation with more than one sinusoidal signal, unwanted components occur at frequencies which are linear combinations of the input frequencies. These components are denoted as intermodulation products.

The weakly nonlinear behavior of analog integrated circuits is caused by the slight curvature of the characteristics of the devices of the circuit around the operating point. This behavior contrasts with strongly nonlinear behavior, where devices such as transistors switch between an on-state and an off-state. Nonlinear behavior, both weakly and strongly nonlinear, are not always unwanted. For example, oscillators and mixers explicitly rely on nonlinearities for a suitable operation. This book is concentrated on weakly nonlinear behavior only. In this way, the majority of continuous-time analog integrated building blocks is covered.
The harmonics or intermodulation products characterize the amount of nonlinearity of a given circuit. Since sinusoidal signals and sums of sinusoids are frequently used as inputs, a sinusoidal steady-state analysis of weakly nonlinear circuits is certainly not restrictive. Such analysis is carried out in the frequency domain.

The familiarity of circuit engineers with linear systems has given rise to many useful insights and design rules for circuits that can be approximated as linear. A circuit engineer is able to derive closed-form expressions for characteristics of a linearized circuit, which he can interpret and use afterwards during the synthesis of the circuit. If the circuit or its simplified schematic is too large to analyze with hand calculations, he can resort to a symbolic analyzer for linear(ized) circuits, but he still remains able to some extent to reason about the linear circuit’s behavior even without explicitly having expressions for the characteristics.

On the other hand, the analysis and synthesis of circuits in which nonlinearities play a role, is difficult. Indeed, for such circuits just a few design rules exist in the analog design world. This has several reasons. First, circuit designers are trained to reason only about linear systems, but not about nonlinear ones. Secondly, it is not easy to analyze nonlinear effects by hand calculations. The most studious designers use Taylor series, but this approach is only feasible in small circuits at low frequencies, with a very small number of nonlinearities. Usually, nonlinear effects are analyzed with tedious time-domain simulations (so-called transient simulations), followed by a Fourier analysis. Other approaches such as the harmonic balance techniques, although very useful, are not (yet) universally used. With both approaches the simulation results are numbers. These numbers can be plotted onto a graph, which can give valuable information, but they do not indicate the fundamental circuit parameters that determine the observed performance. As a result, circuit designers often do not know in which way a circuit can be improved in order to meet the specifications related to nonlinear behavior.

The above problem could be relieved if it were possible to indicate to circuit designers which circuit elements or which effects are mainly responsible for the observed nonlinear behavior. Such insight will be offered in this book for several building blocks of analog integrated circuits. In addition some general concepts of nonlinear behavior of analog integrated circuits will be studied as well.

This book is intended as a guide to learn designers of analog integrated circuits to reason about nonlinear phenomena in weakly nonlinear, continuous-time analog ICs. The required background to read this book is an understanding of analog integrated circuit design. A prior knowledge about theory of nonlinear systems, such as the theory of Volterra series, is not required. The background of Volterra series that is required to understand some essential concepts, is contained in this book.

When browsing through this book, the reader will notice the large number of formulas in this book. When one wants to reason about nonlinear phenomena, then a minimum amount of mathematics is required. It is virtually impossible to write a comprehensible text on nonlinear effects without mathematics. However, no special advanced mathematical techniques are used in this book. Also, the mathematics are explained as clearly as possible, they are interpreted and illustrated with examples, and tedious derivatives are moved to appendices at the end of the book.

This book is devoted to the analysis in the frequency domain of weakly nonlinear circuits. The circuits that are addressed are building blocks of analog integrated circuits, both in bipolar
and CMOS technologies. The emphasis is on getting insight, both in the nonlinearities in the transistors themselves, as in the nonlinear behavior of transistor circuits.

The outline of the text is as follows:

- Chapter 1 presents an overview of the approach that is followed in this book. Further, some assumptions are made about the nonlinear circuits that will be analyzed in this book, and the scope of the book will be outlined.

- In the analog design community, many definitions and keywords are used to characterize the nonlinear behavior of analog circuits in the frequency domain. This basic terminology is described in Chapter 2.

- In order to analyze the nonlinear behavior of a circuit, we need to describe the different nonlinearities in the circuit under consideration with a sufficient accuracy. To this purpose, we will make use of power series expansions of the model equations that describe a nonlinear device. It will turn out that a device such as a transistor consists of several basic nonlinear elements, such as nonlinear conductances, transconductances and capacitors. Each of these elements can be described with a power series. The coefficients of these power series are proportional to the derivatives of the model equations that describe these basic nonlinear elements. These power series coefficients, further in the text referred to as nonlinearity coefficients determine the nonlinear behavior behavior of a circuit. In Chapter 3, these power series are presented together with some simple examples.

- A very useful technique to describe the nonlinear behavior of weakly nonlinear circuits is the Volterra series approach, which is covered in Chapter 4. With Volterra series, it is possible to take into account frequency effects into the calculations of harmonics and intermodulation products. This is absolutely necessary if one wants to study circuits with capacitors, both linear and nonlinear. Further, we will use Volterra series to study some general concepts in nonlinear circuits: the use of feedback, both linear and nonlinear, the exploitation of symmetry for the suppression of even-order or odd-order harmonics, the effect of cascading nonlinear circuits, and pre- and post-distortion will be studied.

- Calculation methods for harmonics and intermodulation products are discussed in Chapter 5. The emphasis is on methods that allow to generate closed-form expressions for harmonics or intermodulation products. Numerical methods will be discussed briefly.

For the generation of closed-form expressions a calculation method can be used that makes use of Volterra series. This method is explained with an example. Derivations can be found in literature [Buss 74, Chua 79b]. Further, an alternative method is developed in this chapter that yields the same results without making use of Volterra series.

With both methods the circuit is analyzed in the frequency domain. In fact, we perform an AC analysis on a circuit in which the nonlinear elements are not only represented by their linearized equivalent: in addition, the second- and third-order nonlinear behavior are taken into account as well.
Despite the availability of different methods to obtain closed-form expressions for harmonics or intermodulation products, the use of these methods for hand calculations is too tedious, even for very small circuits. With a symbolic network analysis program, however, it is possible to automate these hand calculations and to obtain a closed-form expression. Symbolic network analysis programs can compute a closed-form expression for the AC behavior of a linearized circuit as a function of the symbolic small-signal parameters of the circuit and of the complex frequency variable. Modern symbolic analysis programs, such as the program ISAAC [Giel 89, Giel 91], can even generate approximate symbolic expressions. The approximation is made because the exact expression is usually too lengthy, such that it cannot be easily interpreted. An approximation based upon numerical values for the circuit parameters can retain the few dominant terms of an expression, such that the resulting expression becomes interpretable. In Chapter 5 an extension of the program ISAAC is described for the generation of approximate, interpretable expressions for nonlinear distortion.

- The nonlinearity coefficients of the different nonlinearities in a bipolar transistor and a MOS transistor are discussed in Chapters 6 and 7, respectively. Whereas for the bipolar transistor the Gummel-Poon model is still widely used or it forms the basis of more recent models [Mc And 95], the situation for a MOS transistor is more complicated. Due to the rapid scaling of a MOS transistor, effects that were recognized as second-order effects in older technologies, become dominant in modern devices. If these effects are not included in a transistor model or if they are badly modeled, then this may lead to very large errors on the harmonics or intermodulation products. The reason is that the nonlinearity coefficients are proportional to higher-order derivatives. The error on these derivatives tends to increase dramatically when the model equation is inaccurate. In Chapter 7 some shortcomings of widely used transistor models will be discussed. Further, a model for the drain current will be presented that will be used to derive closed-form expressions for the nonlinearity coefficients.

- Apart from the examples that have been used throughout the individual chapters, some more applications are described in Chapter 8. Distortion will be analyzed for the following circuits: a single-transistor amplifier, both a bipolar and a MOS version, a bipolar and a MOS differential pair, a source follower, an emitter follower, a bipolar transistor with emitter degeneration, a common-base bipolar and a common-gate MOS transistor, a bipolar and a MOS current mirror, a Miller-compensated operational amplifier, a bipolar double-balanced mixer, and a CMOS upconverter. Hereby, use will be made of the extension of the program ISAAC to generate closed-form expressions for distortion.

- Instead of computing the nonlinearity coefficients, it is also possible to measure these coefficients. A measurement procedure and measurement results on bipolar transistors are given in Chapter 9.

Finally the authors would like to thank the following persons for their discussions and comments: Peter Kinget from Lucent Technologies, Murray Hill, Yannis Tsividis from the University of Columbia, New York, Stéphane Donnay, Hugo De Man and Luc Dupas from IMEC,
Leuven, Georges Gielen, Walter Daems and Joos Vandewalle from the ESAT Laboratories of the Catholic University of Leuven, Petr Dobrovolny from the University of Brno, Czech Republic, Guang-Ming Yin and Frederic Stubbe from Rockwell Semiconductor Systems, Newport Beach, California, Frank Op 't Eynde from Alcatel Microelectronics, Brussels, Paco Fernández from IMSE-CNM, Seville, Spain, Jan Vanthienen, and, last but not least, Kaat François for her support and patience.

Piet Wambacq
Willy Sansen
TO OUR FAMILIES
Kaat, Lien, and Elli
Hadewych, Katrien, Sara, and Marjan
List of symbols and abbreviations

The SPICE model parameters are not included. They can be found in [Hspi 96, Anto 88].

\( \times \)  
multiplication symbol in numbers, for example \( 1.5 \times 10^{-11} \).

\( a^* \)  
the complex conjugate of the complex number \( a \).

\( \beta_F \)  
transistor "beta": maximum value of ratio between the collector current and the base current of a bipolar transistor.

\( \gamma \)  
body-effect coefficient of a MOS transistor.

\( \varepsilon_{ox} \)  
\( 3.4531 \times 10^{-11} \text{F/m} \), dielectric permittivity of \( SiO_2 \).

\( \varepsilon_{Si} \)  
\( 1.0359 \times 10^{-10} \text{F/m} \), dielectric permittivity of \( SiO_2 \).

\( \lambda \)  
channel-length modulation factor (MOS transistor).

\( \mu \)  
surface mobility (MOS transistor).

\( \phi \)  
surface inversion potential (MOS transistor).

\( \theta \)  
 mobility-reduction coefficient (MOS transistor).

\( \arg(z) \)  
phase of the complex number \( z \).

BJT  
bipolar junction transistor.

\( C \)  
symbol for a capacitor.

\( C_{\mu} \)  
base-collector capacitance (bipolar transistor).

\( C_{\pi} \)  
base-emitter capacitance (bipolar transistor).
\(C_{cs}\) collector-substrate capacitance (bipolar transistor).

\(C_{db}\) bulk-drain capacitance (MOS transistor).

\(C_{gb}\) gate-bulk capacitance (MOS transistor).

\(C_{gd}\) gate-drain capacitance (MOS transistor).

\(C_{gs}\) gate-source capacitance (MOS transistor).

\(C_{sb}\) bulk-source capacitance (MOS transistor).

\(C'_{ox}\) MOS gate oxide capacitance per unit area.

\(C_{ox}\) total MOS gate oxide capacitance.

\(\text{CMRR}\) common-mode rejection ratio.

\(\det(s)\) determinant of the admittance matrix of a linear network as a function of the complex frequency variable \(s\).

\(E_c\) critical electric field (MOS transistor).

\(E_{eff}\) average normal electric field that is experienced by carriers in the inversion layer (MOS transistor).

\(f'(x), \frac{df}{dx}\) two notations for the derivative of \(f\) with respect to \(x\).

\(g, G\) symbols that are used for conductances.

\(\text{GBW}\) gain-bandwidth product.

\(g_m\) transistor transconductance (MOS and bipolar transistor).

\(g_{mb}\) bulk transconductance (MOS transistor).

\(g_o\) transistor output conductance (MOS and bipolar transistor).

\(g_x\) incremental base-emitter conductance (bipolar transistor).

\(\text{H}_n\) \(n\)-th order Volterra operator.

\(HD_2, HD_3\) second- and third-order harmonic distortion.

\(H_n(j\omega_1, j\omega_2, \ldots, j\omega_n)\) \(n\)-dimensional Fourier transform of the \(n\)-th order Volterra kernel, also denoted as \(n\)-th order transfer function.

\(h_n(\tau_1, \tau_2, \ldots, \tau_n)\) \(n\)-th order Volterra kernel (time-domain representation).
i_{OUT} \quad \text{total current through a component (time domain).}

I_{OUT} \quad \text{DC component of the current through a component.}

i_{\text{out}} \quad \text{AC component of the current through a component (time domain). Hence } i_{\text{OUT}} = I_{OUT} + i_{\text{out}}.

I_{\text{out}} \quad \text{phasor of the current through a component in the steady state (sinusoidal excitation).}

i_B, i_b, I_B, I_b \quad \text{total value (time domain), AC value, DC value and phasor respectively of the base current of a bipolar transistor.}

i_C, i_c, I_C, I_c \quad \text{total value (time domain), AC value, DC value and phasor respectively of the collector current of a bipolar transistor.}

i_D, i_d, I_D, I_d \quad \text{total value (time domain), AC value, DC value and phasor respectively of the drain current of a MOS transistor in the triode region.}

i_{\text{DSAT}}, i_{\text{dsat}}, I_{\text{DSAT}}, I_{\text{dsat}} \quad \text{total value (time domain), AC value, DC value and phasor respectively of the drain current of a MOS transistor in saturation.}

I_{KF} \quad \text{forward knee current (bipolar transistor).}

I_{M2}, I_{M3} \quad \text{second- and third-order intermodulation distortion.}

IMFDR \quad \text{intermodulation-free dynamic range.}

IP_{2h}, IP_{3h} \quad \text{second- and third-order intercept point for harmonics.}

IP_{2i}, IP_{3i} \quad \text{second- and third-order intercept point for intermodulation products.}

I_S \quad \text{saturation current (bipolar transistor).}

I_{SE} \quad \text{base-emitter leakage saturation current (bipolar transistor).}

j \quad \sqrt{-1}

k \quad 1.38062 \times 10^{-23} J/K, \text{ Boltzmann's constant.}
nth-order nonlinearity coefficient in the power series expansion of the function that describes the nonlinear relationship between the current and the controlling voltage for either a nonlinear conductance, transconductance or capacitor. The symbol $x$ represents the linearized equivalent of the nonlinear element.

normalized nonlinearity coefficient: $K_{nx}$ divided by $x$.

$m$th-order nonlinearity coefficient in the two-dimensional power series expansion of the function that describes the nonlinear relationship between the current and the controlling voltages $u$ and $v$ for a two-dimensional transconductance. The symbols $g_1$ and $g_2$ represent the coefficients in the power series of the first-order terms in $u$ and $v$, respectively. If the nonlinear relationship is expressed as $i = f(u, v)$, then $K_{m_j g_1 g_2 (m-j) g_2}$ is given by:

$$
\frac{\partial^m f(u, v)}{\partial u^j \partial v^{m-j}} \cdot \frac{1}{j!} \cdot \frac{1}{(m-j)!}
$$

If $j$ or $(m-j)$ are equal to one, then they are usually omitted, like in $K_{g_1 g_2}$.

$m$th-order nonlinearity coefficient in the three-dimensional power series expansion of the function that describes the nonlinear relationship between the current and the controlling voltages $u, v$ and $w$ for a three-dimensional transconductance. The symbols $g_1, g_2$ and $g_3$ represent the coefficients in the power series of the first-order terms in $u, v$ and $w$, respectively. If the nonlinear relationship is expressed as $i = f(u, v, w)$, then $K_{m_j g_1 g_2 (m-j-k) g_3}$ is given by:

$$
\frac{\partial^n f(u, v, w)}{\partial u^j \partial v^k \partial w^{m-j-k}} \cdot \frac{1}{j!} \cdot \frac{1}{k!} \cdot \frac{1}{(m-j-k)!}
$$

Note that, if $j, k$ or $(m-j-k)$ are equal to one, then they are usually omitted, like in $K_{g_1 g_2 g_3}$.

effective channel length of a MOS transistor.
$N_A$  
acceptor concentration in the bulk region of a MOS transistor.

$n_E$  
base-emitter emission coefficient (bipolar transistor).

$n_F$  
forward emission coefficient (bipolar transistor).

$n_i$  
intrinsic carrier concentration ($1.45 \times 10^{10} \text{cm}^{-3}$ at room temperature).

$P_{-1dB}, P_{-3dB}$  
1dB or 3dB compression point.

$q$  
1.6022 $C_e$, elementary charge.

$Q_B$  
majority charge in the neutral base region (bipolar transistor).

$Q_{BO}$  
majority charge in the neutral base region at $v_{BC} = 0V$.

$Q'_I(x)$  
inversion layer charge per unit area at the position $x$ in the channel of a MOS transistor ($0 \leq x \leq L$).

$r, R$  
symbols that are used for resistances; $r = 1/g$ and $R = 1/G$.

$R_B, r_B$  
DC and AC base resistance (bipolar transistor).

$R_{BeX}, r_{BeX}$  
DC and AC extrinsic base resistance (bipolar transistor). In SPICE this is denoted by $RBM$.

$R_{BI}, r_{BI}$  
DC and AC intrinsic base resistance (bipolar transistor).

$RF$  
radio frequency.

$r_o$  
transistor output resistance (bipolar and MOS transistor).

$r_\pi$  
icremental base-emitter resistance (bipolar transistor).

$s$  
complex frequency variable (Laplace transform variable).

$T$  
absolute temperature (in degrees Kelvin).

$TF_{i_1 \rightarrow \text{output}}$  
transfer function from a current source $i_1$ to the output of interest, which can be a node voltage or a current through a circuit element.
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<th>Definition</th>
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<tr>
<td><strong>THD</strong></td>
<td>total harmonic distortion.</td>
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<tr>
<td>$t_{ox}$</td>
<td>gate-oxide thickness (MOS transistor).</td>
</tr>
<tr>
<td>$V_{AF}$</td>
<td>(forward) Early voltage (bipolar transistor).</td>
</tr>
<tr>
<td>$v_{CONTR}$</td>
<td>total value of a voltage that controls a nonlinear circuit element (time domain).</td>
</tr>
<tr>
<td>$V_{CONTR}$</td>
<td>DC component of the voltage that controls a nonlinear circuit element.</td>
</tr>
<tr>
<td>$v_{contr}$</td>
<td>AC component (time domain) of the voltage that controls a nonlinear circuit element (time domain). Hence $v_{CONTR} = V_{CONTR} + v_{contr}$.</td>
</tr>
<tr>
<td>$V_{contr,m,n}$</td>
<td>phasor of the component at the frequency $</td>
</tr>
<tr>
<td>$v_{pq}$</td>
<td>AC component (time domain) of the difference between the voltage at node $p$ and $q$: $v_{pq} = v_p - v_q$.</td>
</tr>
<tr>
<td>$V_{pq,m,n}$</td>
<td>phasor of the component at the frequency $</td>
</tr>
<tr>
<td>$v_{DSAT}$</td>
<td>drain-source saturation voltage (MOS transistor).</td>
</tr>
<tr>
<td>$v_{sat}$</td>
<td>thermal velocity of carriers, also denoted as saturation velocity.</td>
</tr>
<tr>
<td>$V_{TO}$</td>
<td>zero-bias gate-source extrapolated threshold voltage of a MOS transistor.</td>
</tr>
<tr>
<td>$V_T$</td>
<td>gate-source extrapolated threshold voltage of a MOS transistor.</td>
</tr>
<tr>
<td>$V_i$</td>
<td>the thermal voltage $kT/q$ (25.86 mV at room temperature).</td>
</tr>
<tr>
<td>$W$</td>
<td>effective channel width of a MOS transistor.</td>
</tr>
<tr>
<td>$\omega, \omega_1, \omega_2$</td>
<td>pulsation ($= 2 \pi \times$ frequency) ($rad/sec$).</td>
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