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An Invitation to Algebraic Geometry

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Notes for the Second Printing

The second printing of this book corrects the many typos and errors that were brought to our attention by readers from around the world. We have also added a few exercises and clarified parts of the text. We are grateful to all the readers who have helped improve our book, but owe particular thanks to Brian Conrad, Sándor Kovács, Grisha Stewart, and especially to Rahim Zaare Nahandi of the University of Tehran, who is engaged in translating this volume into Persian.

Karen E. Smith
Berkeley, CA, USA
March 2003

Preface

These notes grew out of a course at the University of Jyväskylä in January 1996 as part of Finland's new graduate school in mathematics. The course was suggested by Professor Kari Astala, who asked me to give a series of ten two-hour lectures entitled "Algebraic Geometry for Analysts." The audience consisted mainly of two groups of mathematicians: Ph.D. students from the Universities of Jyväskylä and Helsinki, and mature mathematicians whose research and training were quite far removed from algebra. Finland has a rich tradition in classical and topological analysis, and it was primarily in this tradition that my audience was educated, although there were representatives of another well–known Finnish school, mathematical logic.

I tried to conduct a course that would be accessible to everyone, but that would take participants beyond the standard course in algebraic geometry. I wanted to convey a feeling for the underlying algebraic principles of algebraic geometry. But equally important, I wanted to explain some of algebraic geometry's major achievements in the twentieth century, as well as some of the problems that occupy its practitioners today. With such ambitious goals, it was necessary to omit many proofs and sacrifice some rigor.

In light of the background of the audience, few algebraic prerequisites were presumed beyond a basic course in linear algebra. On the other hand, the language of elementary point-set topology and some basic facts from complex analysis were used freely, as was a passing familiarity with the definition of a manifold.

My sketchy lectures were beautifully written up and massaged into this text by Lauri Kahanpää and Pekka Kekäläinen. This was a Herculean effort, no less because of the excellent figures Lauri created with the computer. Extensive revisions to the Finnish text were carried out together with Lauri and Pekka; later Will Traves joined in to help with substantial revisions to the English version. What finally resulted is this book, and it would not have been possible without the valuable contributions of all members of our four-author team.

This book is intended for the working or the aspiring mathematician who is unfamiliar with algebraic geometry but wishes to gain an appreciation of its foundations and its goals with a minimum of prerequisites. It is not intended to compete with such comprehensive introductions as Hartshorne's or Shafarevich's texts, to which we freely refer for proofs and rigor. Rather, we hope that at least some readers will be inspired to undertake more serious study of this beautiful subject. This book is, in short, An Invitation to Algebraic Geometry.

Karen E. Smith
Jyväskylä, Finland
August 1998

Acknowledgments

The notes of Ari Lehtonen, Jouni Parkkonen, and Tero Kilpeläinen complemented those of authors Lauri and Pekka in producing a typed version of the original lectures. Comments of Osmo Pekonen, Ari Lehtonen, and Lassi Kurittu then helped eradicate most of the misprints and misunderstandings marring the first draft, and remarks of Bill Fulton later helped improve the manuscript. Artistic advice from Virpi Kauko greatly improved the pictures, although we were able to execute her suggestions only with the help of Ari Lehtonen's prize-winning Mathematica skills. Computer support from Ari and from Bonnie Freidman at MIT made working together feasible in Jyväskylä and in the US despite different computer systems. The suggestions of Manuel Blickle, Mario Bonk, Bill Fulton, Juha Heinonen, Eero Hyry, and Irena Swanson improved the final exposition. We are especially grateful to Eero for comments on the Finnish version; as one of the few algebraic geometers working in Finland, he advised us on the choices we made regarding mathematical terminology in the Finnish language. The Finnish craftwork of Liisa Heinonen provided instructive props for the lectures, most notably the traditional Christmas Blowup, whose image appears inside the cover of this book. Cooperation with Ari Lehtonen was crucial in creating the photograph. The lectures were hosted by the University of Jyväskylä mathematics department, and we are indebted to the chairman, Tapani Kuusalo, for making them possible. Finally, author Karen acknowledges the patience of her daughter Sanelma during the final stages of work on this project, and the support of her husband and babysitter, Juha Heinonen.

Karen E. Smith
Ann Arbor, Michigan, USA
July 2000

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$\mathbf{PGL}(n, \mathbb{C})$ automorphism group of $\mathbb{P}^{n-1}$
$\mathbb{R}$ real numbers
$\tilde{R}$ sheaf associated to $\mathrm{Spec}(R)$
$\mathcal{R}(U)$ sections of a sheaf $\mathcal{R}$ over an open set U
$X \dashrightarrow Y$ rational map from X to Y
$\mathrm{Sec}\, X$ secant variety to X
$\mathbf{SL}(n, \mathbb{C})$ group of $n \times n$ complex matrices with determinant 1
$\mathrm{Spec}(R)$ spectrum of a ring R
$\Sigma_{m,n}$ Segre mapping
$\mathrm{Sing}\, V$ singular locus of V
$\mathrm{Tan}\, X$ tangent variety to X
T_pV tangent space to V at the point p
TV total tangent bundle to V
Θ_X sheaf of sections of the tangent bundle
$\mathbf{U}(n)$ group of unitary $(n \times n)$-matrices
$\bar{V}$ projective closure of V
$\mathbb{V}(\{F_i\})$ common zeros of the polynomials F_i
ν_d Veronese mapping of degree d
$\mathbb{Z}$ integers