Part II

Imperfect Bifurcation of Symmetric Systems

Symmetry can be found literally everywhere as was introduced, e.g., by Weyl, 1952 [191], Stewart and Golubitsky, 1992 [169], and Rosen, 1995 [157]. One might have been amazed at the symmetry and orderliness of the honeycomb, which is made up of a number of hexagons arranged in order. At the expense of beauty and orderliness, symmetric systems often undergo complex physical behavior that is called "pattern selection" or "pattern formation" (see, e.g., Chadam et al., 1996 [23]). In fluid mechanics, nonlinear mathematics, and so on, it is well-known that patterns are selected or formed through recursive bifurcation which "breaks" symmetry. The Couette–Taylor flow in a hollow cylinder, which is a rotating annular of fluid, displays wave patterns with a variety of symmetries through pattern selection (e.g., Taylor, 1923 [171]). The convective motion of fluid in the so-called Benard problem often displays a regularly arrayed, hexagonal, cellular structure (e.g., Koschmieder, 1974 [116]).

Symmetry is described by means of a group. Moreover, bifurcation structures near singular points can be investigated theoretically by group-theoretic bifurcation theory in nonlinear mathematics. We can find a group G that labels the symmetry of the system under consideration, and a hierarchy of subgroups

$$G \to G_1 \to G_2 \to \cdots,$$

that characterizes the recursive occurrence of bifurcations. Here $\to$ denotes a bifurcation, and G_i $(i = 1, 2, \ldots)$ stand for the nesting subgroups of G that label the reduced symmetry of the bifurcated solutions. Knowledge of such a hierarchy will be vital in the complete description of recursive bifurcation behavior.

In the modeling of the bifurcation phenomena of a symmetric system, we need to find the particular group that labels the symmetry of the system under consideration because the rule of the bifurcation of this system, such as the hierarchy of subgroups presented above, is dependent on the group. In order to avoid sophisticated mathematical concepts, we focus in this part, Part II, on the apparent geometrical symmetry labeled by the simplest groups, the dihedral and cyclic groups. The bifurcation of systems with various kinds of symmetries labeled by other groups will be studied in the next part, Part III.

Multiple critical points, where more than one eigenvalue of the Jacobian matrix simultaneously vanishes, appear in symmetric systems. The critical points which appear in these systems, accordingly, consist of two major types, simple critical points and multiple critical points. This shows a sharp contrast with the case of structures without symmetries or with a reflectional symmetry, where only simple critical points appear generically.

Part II consists of five chapters. In Chapter 7 a brief account of group-theoretic bifurcation theory is presented as a basic mathematical tool to describe the bifurcation behavior of a symmetric system. In Chapter 8, by elementary calculations based on the theory in Chapter 7, the rules of the perfect and imperfect bifurcation behavior of systems with dihedral or cyclic group symmetries are obtained. These rules are put to use in the description of the perfect bifurcation behavior of regular polygonal truss domes. In Chapter 9 the procedure to obtain the critical initial imperfection vector is presented based on the concept of the group equivariance of an imperfect system. In Chapter 10 the probabilistic variation of critical loads is formulated for initial imperfections subject to a multivariate normal distribution. In Chapter 11 the perfect and imperfect bifurcation behaviors of realistic systems with dihedral symmetries are investigated.