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Topics in Industrial Mathematics

Case Studies and Related Mathematical Methods

by

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To Renate Neunzert and Azra Siddiqi

without whose patience and cooperation this work would not have been possible.

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Preface

Industrial Mathematics is a relatively recent discipline. It is concerned primarily with transforming technical, organizational and economic problems posed by industry into mathematical problems; “solving” these problems by approximative methods of analytical and/or numerical nature; and finally reinterpreting the results in terms of the original problems. In short, industrial mathematics is modelling and scientific computing of industrial problems.

Industrial mathematicians are bridge-builders: they build bridges from the field of mathematics to the practical world; to do that they need to know about both sides, the problems from the companies and ideas and methods from mathematics. As mathematicians, they have to be generalists. If you enter the world of industry, you never know which kind of problems you will encounter, and which kind of mathematical concepts and methods you will need to solve them. Hence, to be a good “industrial mathematician” you need to know a good deal of mathematics as well as ideas already common in engineering and modern mathematics with tremendous potential for application. Mathematical concepts like wavelets, pseudorandom numbers, inverse problems, multigrid etc., introduced during the last 20 years have recently started entering the world of real applications.

Industrial mathematics consists of modelling, discretization, analysis and visualization. To make a good model, to transform the industrial problem into a mathematical one such that you can trust the prediction of the model is no easy task. One needs plenty of experience because modelling is mainly learnt by doing. A nice approach would be to pose real-world problems to the students who should work on them under the guidance of an experienced modeler. In international programmes in Kaiserslautern, “modelling seminars” are organized each semester along these lines. They are proving an important tool for the education of industrial mathematicians. The problems are mainly supplied by an “Institute for Industrial Mathematics” which cooperates with industry on a large scale, doing about 40 different projects every year. This institute is a very important source of interesting problems. But not every university has such a source. How do others get appropriate problems? Again, by searching them, where they are—in industry. Staff members have to visit companies, discuss their problems—they will find a variety of good projects.

This book is designed to help the beginners, to show what we have experienced during our interaction with industry and teaching industrial mathematics. It tries to teach modelling by reading. It may not be the best solution—learning by doing is clearly preferable. We have, however, tried to maintain the flavour: first, by presenting five case studies and then adding some background material related to the theories used for the case studies.

The case studies which make up the first chapter of the book have been taken from a modelling seminar in Kaiserslautern and handle problems of molecular alignment in drug design, acoustic identification of vehicles, the security of air bag sensors, quality control of fabrics and fatigue life analysis. The subsequent chapters provide the reader with mathematical concepts and methods which are essential for a proper

analysis of these models and for exploration of related new areas.

For example, a problem of fatigue life analysis dealing with the estimation of the lifetime of critical car components is presented in Chapter 1, while a mathematical formulation based on the concept of hysteresis is given in chapter 6. Airbag sensors need Maxwell's equations whose basis and relevant literature is given in Chapter 3. Optimization is needed in drug design and the acoustic identification of vehicles – Chapter 2 discusses some important algorithms in continuous optimization. Random numbers and so-called Monte Carlo methods help to evaluate very complex integrals (as in drug design) and to solve high-dimensional kinetic equations needed for nuclear reactors, space flight, semiconductors. Chapter 4 is devoted to these methods. Image processing is an emerging field, where a whole bunch of new mathematical ideas are used; our problem deals with quality control of fabrics and uses the fundamental concepts of multiscale analysis. But other methods like wavelets, fractals, energy model, etc. may also be equally important and we describe some of them in Chapter 5.

In the appendices, we have provided discussion on certain topics which are essential for understanding of the main text as well as some results which could not find an appropriate place in a particular chapter. At the end of each chapter, we have given some problems, some of which may lead to research problems, especially in Chapters 3 and 5. Hints of some of these problems are mentioned there. At the end, we have provided an extensive bibliography.

The book addresses several types of readers. We hope it to be useful for all those who have genuine curiosity to know about Industrial Mathematics. It is intended as a handy manual of Mathematical Methods for current industrial and technological problems which may be very useful for engineers and physicists. It is also intended to serve as a lucid commentary on most applied methods which are likely to attract more attention in years to come. Finally, it can be used for a course on Mathematical Methods of current real life problems/industrial problems at graduate and advanced undergraduate levels. A deeper insight, if needed, can be obtained through updated and appropriate references mentioned in the text. Proof of theorems like Theorems 3.2, 3.3 and 6.3 may be omitted by the readers who are not interested in a rigorous analysis.

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