Division Algebras:
Octonions,
Quaternions,
Complex Numbers
and the Algebraic Design of Physics

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Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics

by

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Preface

I don't know who Gigerenzer is, but he wrote something very clever that I saw quoted in a popular glossy magazine:

"Evolution has tuned the way we think to frequencies of co-occurances, as with the hunter who remembers the area where he has had the most success killing game."

This sanguine thought explains my obsession with the division algebras. Every effort I have ever made to connect them to physics - to the design of reality - has succeeded, with my expectations often surpassed. Doubtless this strong statement is colored by a selective memory, but the kind of game I sought, and still seek, seems to frowst about this particular watering hole in droves. I settled down there some years ago and have never felt like leaving.

This book is about the beasts I selected for attention (if you will, to render this metaphor politically correct, let's say I was a nature photographer), and the kind of tools I had to develop to get the kind of shots I wanted (the tools that I found there were for my taste overly abstract and theoretical). Half of this book is about these tools, and some applications thereof that should demonstrate their power. The rest is devoted to a demonstration of the intimate connection between the mathematics of the division algebras and the Standard Model of quarks and leptons with $U(1) \times SU(2) \times SU(3)$ gauge fields, and the connection of this model to 10-dimensional spacetime implied by the mathematics.

If you understand what I have written to this point, then there is a good chance you have the background to understand this book. As to agreeing with its philosophical premises, you probably won't. I've never yet found two mathematicians or physicists who agree on the connection between mathematics and physics, and its depth. However, such agreement is not required to appreciate the material presented.

I owe thanks to several people: to my wife, Suzanne Young of Harvard, whose work in archaeometry compelled me in some weird way to write the book; to my thesis advisor, Hugh Pendleton of Brandeis, for his advice, interest, and forbearance; and to Rafal Ablamowicz, Martin Cederwall, Pertti Lounesto, Corinne Manogue, Ian Porteous, and Tony Smith, all of whom at my instigation converged on Göteborg, Sweden, this last January (1994) to talk about division algebras, Clifford algebras, and physics. Each played a crucial role at this meeting, and the breadth and depth of their insights has greatly enriched this monograph.

Geoffrey Dixon, Waltham, Massachusetts, 21 February 1994.