PART III
HAMILTONIAN MECHANICS

Hamiltonian mechanics is geometry in phase space. Phase space has the structure of a symplectic manifold. The group of symplectic diffeomorphisms acts on phase space. The basic concepts and theorems of hamiltonian mechanics (even when formulated in terms of local symplectic coordinates) are invariant under this group (and under the larger group of transformations which also transform time).

A hamiltonian mechanical system is given by an even-dimensional manifold (the "phase space"), a symplectic structure on it (the "Poincaré integral invariant") and a function on it (the "hamiltonian function"). Every one-parameter group of symplectic diffeomorphisms of the phase space preserving the hamiltonian function is associated to a first integral of the equations of motion.

Lagrangian mechanics is contained in hamiltonian mechanics as a special case (the phase space in this case is the cotangent bundle of the configuration space, and the hamiltonian function is the Legendre transform of the lagrangian function).

The hamiltonian point of view allows us to solve completely a series of mechanical problems which do not yield solutions by other means (for example, the problem of attraction by two stationary centers and the problem of geodesics on the triaxial ellipsoid). The hamiltonian point of view has even greater value for the approximate methods of perturbation theory (celestial mechanics), for understanding the general character of motion in complicated mechanical systems (ergodic theory, statistical mechanics) and in connection with other areas of mathematical physics (optics, quantum mechanics, etc.).