

PART II

LAGRANGIAN MECHANICS

Lagrangian mechanics describes motion in a mechanical system by means of the configuration space. The configuration space of a mechanical system has the structure of a differentiable manifold, on which its group of diffeomorphisms acts. The basic ideas and theorems of lagrangian mechanics are invariant under this group,²⁵ even if formulated in terms of local coordinates.

A lagrangian mechanical system is given by a manifold (“configuration space”) and a function on its tangent bundle (“the lagrangian function”).

Every one-parameter group of diffeomorphisms of configuration space which fixes the lagrangian function defines a conservation law (i.e., a first integral of the equations of motion).

A newtonian potential system is a particular case of a lagrangian system (the configuration space in this case is euclidean, and the lagrangian function is the difference between the kinetic and potential energies).

The lagrangian point of view allows us to solve completely a series of important mechanical problems, including problems in the theory of small oscillations and in the dynamics of a rigid body.

²⁵ And even under larger groups of transformations, which also affect time.