

SEMINARS IN MATHEMATICS
V. A. STEKLOV MATHEMATICAL INSTITUTE, Leningrad

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МАТЕМАТИЧЕСКОГО ИНСТИТУТА ИМ. В.А. СТЕКЛОВА АН СССР

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MATHEMATICAL PROBLEMS IN WAVE PROPAGATION THEORY

Edited by V. M. Babich

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PREFACE

The papers comprising this collection are directly or indirectly related to an important branch of mathematical physics – the mathematical theory of wave propagation and diffraction.

The paper by V. M. Babich is concerned with the application of the parabolic-equation method (of Academician V. A. Fok and M. A. Leontovich) to the problem of the asymptotic behavior of eigenfunctions concentrated in a neighborhood of a closed geodesic in a Riemannian space. The techniques used in this paper have been found useful in solving certain problems in the theory of open resonators.

The topic of G. P. Astrakhantsev's paper is similar to that of the paper by V. M. Babich. Here also the parabolic-equation method is used to find the asymptotic solution of the elasticity equations which describes Love waves concentrated in a neighborhood of some surface ray.

The paper of T. F. Pankratova is concerned with finding the asymptotic behavior of the eigenfunctions of the Laplace operator from the exact solution for the surface of a triaxial ellipsoid and the region exterior to it.

The first three papers of B. G. Nikolaev are somewhat apart from the central theme of the collection; they treat the integral transforms with respect to associated Legendre functions of first kind and their applications. Examples of such applications are the use of this transform for the solution of integral equations with symmetric kernels and for the solution of certain problems in the theory of electrical prospecting.

The two papers by B. G. Nikolaev are devoted to describing one of the possible approaches to the solution of stationary problems related to the diffraction of waves by angular plane regions. Examples considered are the Neumann problem for the exterior of a wedge-shaped region and the Neumann problem for a half space divided by an inclined boundary into two angular regions with different wave-propagation speeds.

The paper by A. I. Lanin is a continuation of the joint work of V. S. Buldyrev and A. I. Lanin on interference waves in diffraction problems for the cylinder and sphere. The intensities of interference-type waves and of noninterfering waves of geometrical optics are computed on the basis of tabulated results for the special functions $G_m(\gamma)$ and $\Gamma_m(\gamma)$, which describe the interference fields. The behavior of the field in a neighborhood of a limiting ray for the sphere and planar case is compared.

In the paper "Calculation of the wave fields for multiple waves near the points of origin" by N. S. Smirnova the principal parts of the displacement fields are studied for reflected waves and for multiple head waves of arbitrary type which arise in a system of n plane-parallel elastic layers. Attention is focused on obtaining computational formulas which describe the displacement fields near the point of origin of the head wave. A number of particular examples are considered.

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