

A. J. Chorin and J. E. Marsden

A Mathematical Introduction to Fluid Mechanics



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A. J. Chorin and J. E. Marsden
Department of Mathematics
University of California
Berkeley, California 94720
USA

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Preface

These notes are based on a one-quarter (i.e. very short) course in fluid mechanics taught in the Department of Mathematics of the University of California, Berkeley during the Spring of 1978. The goal of the course was not to provide an exhaustive account of fluid mechanics, nor to assess the engineering value of various approximation procedures. The goals were: (i) to present some of the basic ideas of fluid mechanics in a mathematically attractive manner (which does not mean "fully rigorous"); (ii) to present the physical background and motivation for some constructions which have been used in recent mathematical and numerical work on the Navier-Stokes equations and on hyperbolic systems; (iii) to interest some of the students in this beautiful and difficult subject.

The notes are divided into three chapters. The first chapter contains an elementary derivation of the equations; the concept of vorticity is introduced at an early stage. The second chapter contains a discussion of potential flow, vortex motion, and boundary layers. A construction of boundary layers using vortex sheets and random walks is presented; it is hoped that it helps to clarify the ideas. The third chapter contains an analysis of one-dimensional gas

flow, from a mildly modern point of view. Weak solutions, Riemann problems, Glimm's scheme, and combustion waves are discussed.

The style is informal and no attempt was made to hide the authors' biases and interests. Moreover, references have been very limited and are by no means exhaustive. We list below some general references that have been useful for us and which contain fairly extensive bibliographies. References to specific points are made directly in the text.

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